إجابات أسئلة الدرس

التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ) $\int (١ - ٢\text{س})(\text{س} - \text{س}^{٢})^{٤}\,\text{دس}$

ب) $\int ٦\text{س}^{٢}\sqrt[٥]{(٢\text{س}^{٣} - ٢)^{٢}}\,\text{دس}$

جـ) $\int (٢\text{س} - ٣\text{س}^{٢})\,\text{قا}^{٢}(\text{س}^{٢} - \text{س}^{٣})\,\text{دس}$

د) $\int \dfrac{٣\text{س} - ٩}{(\text{س}^{٢} - ٦\text{س})^{٢}}\,\text{دس}$

الحل

أ) $\int (١ - ٢\text{س})(\text{س} - \text{س}^{٢})^{٤}\,\text{دس}$.

$\text{ص} = \text{س} - \text{س}^{٢} \Leftarrow \dfrac{\text{دص}}{\text{دس}} = ١ - ٢\text{س} \Leftarrow \dfrac{\text{دص}}{١ - ٢\text{س}} = \text{دس}$

$\int \cancel{(١ - ٢\text{س})}\,\text{ص}^{٤}\,\dfrac{\text{دص}}{\cancel{١ - ٢\text{س}}} = \int \text{ص}^{٤}\,\text{دص}$

$= \dfrac{\text{ص}^{٥}}{٥} + \text{ج} = \dfrac{(\text{س} - \text{س}^{٢})^{٥}}{٥} + \text{ج}$

ب) $\int ٦\text{س}^{٢}\sqrt[٥]{(٢\text{س}^{٣} - ٢)^{٢}}\,\text{دس}$

$\text{ص} = ٢\text{س}^{٣} - ٢ \Leftarrow \dfrac{\text{دص}}{\text{دس}} = ٦\text{س}^{٢} \Leftarrow \dfrac{\text{دص}}{٦\text{س}^{٢}} = \text{دس}$

$\int \cancel{٦\text{س}^{٢}}\sqrt[٥]{\text{ص}^{٢}}\,\dfrac{\text{دص}}{\cancel{٦\text{س}^{٢}}} = \int \text{ص}^{\frac{٢}{٥}}\,\text{دص}$

$$= \frac{\text{ص}^{\frac{4}{5}+1}}{\frac{4}{5}+1} + \text{جـ} = \frac{\text{ص}^{\frac{9}{5}}}{\frac{9}{5}} + \text{جـ}$$

$$= \frac{5}{9}\sqrt[5]{\text{ص}^{9}} + \text{جـ}$$

$$= \frac{5}{9}\sqrt[5]{(2\text{س}^{3}-2)^{9}} + \text{جـ}$$

جـ) $\int (2\text{س} - 3\text{س}^{2})\,\text{قا}^{2}(\text{س}^{3} - \text{س}^{2})\,\text{دس}$

$$\text{ص} = \text{س}^{3} - \text{س}^{2} \Leftarrow \frac{\text{دص}}{\text{دس}} = 3\text{س}^{2} - 2\text{س}$$

$$\frac{\text{دص}}{3\text{س}^{2} - 2\text{س}} = \text{دس}\ .$$

$$\int \cancel{(2\text{س} - 3\text{س}^{2})}^{\,-1}\,\text{قا}^{2}\,\text{ص}\ \frac{\text{دص}}{\cancel{3\text{س}^{2} - 2\text{س}}}$$

$$\int -\text{قا}^{2}\,\text{ص}\cdot\text{دص} = -\text{ظا}\,\text{ص} + \text{جـ}$$

$$= -\text{ظا}(\text{س}^{3} - \text{س}^{2}) + \text{جـ}$$

د) $\int \frac{3\text{س} - 9}{(\text{س}^{2} - 6\text{س})^{2}}\,\text{دس}$

$$\text{ص} = \text{س}^{2} - 6\text{س} \Leftarrow \frac{\text{دص}}{\text{دس}} = 2\text{س} - 6 \Leftarrow$$

$$\frac{\text{دص}}{2\text{س} - 6} = \text{دس}\ .$$

$$\int \frac{3\text{س} - 9}{\text{ص}^{2}} \times \frac{\text{دص}}{2\text{س} - 6} =$$

$$\int 3\cancel{(\text{س} - 3)} \times \text{ص}^{-2} \times \frac{\text{دص}}{2\cancel{(\text{س} - 3)}} = .$$

$$\frac{3}{2}\,\frac{\text{ص}^{-2+1}}{-2+1} + \text{جـ} = \frac{3}{2}\,\frac{\text{ص}^{-1}}{-1} + \text{جـ}$$

$$-\frac{3}{2\text{ص}} + \text{جـ} = -\frac{3}{2(\text{س}^{2} - 6\text{س})} + \text{جـ}$$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{ءس}$

ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{ءس}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{ءس}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{ءس}$

الحل

أ) $\int \sqrt[3]{(\text{٣س}-٢)^{٢}}\ \text{ءس} = \int (\text{٣س}-٢)^{\frac{٢}{٣}}\ \text{ءس} = \dfrac{(\text{٣س}-٢)^{\frac{٢}{٣}+١}}{(\frac{٢}{٣}+١)\times ٣} + \text{ج}$

$= \dfrac{(\text{٣س}-٢)^{\frac{٥}{٣}}}{\cancel{\frac{٥}{٣}\times ٣}} + \text{ج} = \dfrac{١}{٥}\sqrt[3]{(\text{٣س}-٢)^{٥}} + \text{ج}$

ب) $\int (\text{س}-١)(\text{٢س}^{٢}-\text{٤س}+١)^{٥}\ \text{ءس}$

$\text{ص} = \text{٢س}^{٢}-\text{٤س}+١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س}-٤$

$\dfrac{\text{ءص}}{\text{٤س}-٤} = \text{ءس} \Leftarrow \dfrac{\text{ءص}}{٤(\text{س}-١)} = \text{ءس}$

$\int \cancel{(\text{س}-١)}\ \text{ص}^{٥} \times \dfrac{\text{ءص}}{٤\cancel{(\text{س}-١)}} = \dfrac{١}{٤}\int \text{ص}^{٥}\ \text{ءص}$

$= \dfrac{\text{ص}^{٦}}{٦\times ٤} + \text{ج} = \dfrac{(\text{٢س}^{٢}-\text{٤س}+١)^{٦}}{٢٤} + \text{ج}$

جـ) $\int ٢\ \text{قا}^{٢}(٢-\text{س})\ \text{ءس} =$

$٢\ \dfrac{\text{ظا}(٢-\text{س})}{-١} + \text{ج}$

$-٢\ \text{ظا}(٢-\text{س}) + \text{ج}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤}+١)\ \text{ءس}$

$\text{ص} = \text{س}^{٤}+١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س}^{٣} \Leftarrow$

$\dfrac{\text{ءص}}{\text{٤س}^{٣}} = \text{ءس}$

$\int ٢\cancel{\text{س}^{٣}}\ \text{جا ص}\ \dfrac{\text{ءص}}{٤\cancel{\text{س}^{٣}}} =$

$\int \dfrac{١}{٢}\ \text{جا ص ءص} = -\dfrac{١}{٢}\ \text{جتا ص} + \text{ج}$

$= -\dfrac{١}{٢}\ \text{جتا}(\text{س}^{٤}+١) + \text{ج}$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{٠}^{٢} \sqrt{٤\text{س} + ١}\ \text{ءس}$

ب) $\int_{-١}^{-١} ٣\text{س}^{٢}(\text{س}^{٣} - ١)^{٣}\ \text{ءس}$

جـ) $\int_{١}^{\cdot} ٢\text{س} \sqrt[٣]{\text{س}^{٢} - ١}\ \text{ءس}$

د) $\int_{١}^{٢} \frac{٣ - ٢\text{س}}{(\text{س}^{٢} - ٣\text{س})^{٢}}\ \text{ءس}$

الحل

أ) $\int_{٠}^{٢} \sqrt{٤\text{س}+١}\ \text{ءس} = \int_{٠}^{٢} (٤\text{س}+١)^{\frac{١}{٢}} \cdot \text{ءس}$

$$= \left[\frac{(٤\text{س}+١)^{\frac{١}{٢}+١}}{٤ \times (\frac{١}{٢}+١)} \right]_{٠}^{٢} = \left[\frac{(٤\text{س}+١)^{\frac{٣}{٢}}}{٤ \times \frac{٣}{٢}} \right]_{٠}^{٢}$$

$$\left[\frac{\sqrt{(٤\text{س}+١)^{٣}}}{٦} \right]_{٠}^{٢}$$

$$\frac{١}{٦}\left[\sqrt{(٤ \times ٢ + ١)^{٣}} - \sqrt{(٤ \times ٠ + ١)^{٣}} \right]$$

$$\frac{١}{٦}\left(\sqrt{٩^{٣}} - \sqrt{١^{٣}} \right)$$

$$\frac{1}{6}(27-1)$$

$$\frac{1}{6}\times 26 = \frac{13}{3}$$

ب) $\int_{-1}^{1} 3\text{س}^2(\text{س}^3-1)^3\, \text{دس} = \text{صفر}$

جـ) $\int_{1}^{0} 2\text{س}\,\sqrt[3]{\text{س}^2-1}\,.\text{دس} =$

$$\int_{1}^{0} 2\text{س}\,(\text{س}^2-1)^{\frac{1}{3}}\,\text{دس}$$

$$\text{ص} = \text{س}^2 - 1 \Leftarrow \frac{\text{دص}}{\text{دس}} = 2\text{س} \Leftarrow \text{دس} = \frac{\text{دص}}{2\text{س}}$$

$$\int_{1}^{0} \cancel{2\text{س}}\;\text{ص}^{\frac{1}{3}}\,\frac{\text{دص}}{\cancel{2\text{س}}} = \int_{1}^{0} \text{ص}^{\frac{1}{3}}\,\text{دص}$$

$$\left[\frac{\text{ص}^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{1}^{0} = \left[\frac{\text{ص}^{\frac{4}{3}}}{\frac{4}{3}}\right]_{1}^{0}$$

$$\frac{3}{4}\left[\sqrt[3]{(\text{س}^2-1)^4}\right]_{1}^{0} = \frac{3}{4}\left(\sqrt[3]{(0^2-1)^4} - \sqrt[3]{(1-1)^4}\right)$$

$$\frac{٣}{٤}\left(\sqrt[٣]{١} - \sqrt[٣]{٠}\right)$$

$$\frac{٣}{٤}\left(١ - ٠\right)$$

$$\frac{٣}{٤} \times ١ = \frac{٣}{٤}$$

د) $\int_{١}^{٢} \frac{٢س - ٣}{(س^{٢} - ٣س)^{٢}}\, دس = \int_{١}^{٢} (٢س - ٣)(س^{٢} - ٣س)^{-٢} \cdot دس$

$ص = س^{٢} - ٣س \Leftarrow \frac{دص}{دس} = ٢س - ٣ \Leftarrow دس = \frac{دص}{٢س - ٣}$

$\int_{١}^{٢} \cancel{(٢س - ٣)}\, ص^{-٢} \div \frac{دص}{\cancel{٢س - ٣}} = \int_{١}^{٢} ص^{-٢}\, دص =$ (١-)

$\left[\frac{-ص^{-٢+١}}{-٢+١}\right]_{١}^{٢} = \left[\frac{-ص^{-١}}{-١}\right]_{١}^{٢} = \left[\frac{١}{ص}\right]_{١}^{٢}$

$\left[\frac{١}{س^{٢} - ٣س}\right]_{١}^{٢} = \frac{١}{٢^{٢} - ٣\times٢} - \frac{١}{١^{٢} - ٣\times١} = \frac{١}{-٢} - \frac{١}{-٢}$

$= \frac{-١}{٢} + \frac{١}{٢} =$ صفر

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-٢}^{٣} ٣س^{٢}\, قَ(س^{٣})\, دس$

الحل

$ص = س^{٣} \Leftarrow \frac{دص}{دس} = ٣س^{٢} \Leftarrow دس = \frac{دص}{٣س^{٢}}$

$\int_{-٢}^{٣} \cancel{٣س^{٢}}\, قَ(ص) \frac{دص}{\cancel{٣س^{٢}}} = \int_{-٢}^{٣} قَ(ص) \cdot دص =$

$ق(ص)\Big]_{-٢}^{٣} = ق(س^{٣})\Big]_{-٢}^{٣}$

$ق(٣^{٣}) - ق((-٢)^{٣}) = ق(٢٧) - ق(-٨) = -٦ - ٥$

$= -١١$

٥) إذا علمت أن $\int_{٥}^{٢}$ ق(س) دس = ٣، فجد قيمة التكامل الآتي: $\int_{-١}^{٢}$ ٨س ق(س^{٢} + ١) دس

الحل

$ص = س^{٢} + ١ \Leftarrow \frac{دص}{دس} = ٢س \Leftarrow \frac{دص}{٢س} = دس$

$\int_{-١}^{٢} ٨\cancel{س}\, ق(ص)\, \frac{دص}{٢\cancel{س}} = \int_{-١}^{٢} ٤\, ق(ص)\, دص$

عندما $س = -١ \leftarrow ص = (-١)^{٢} + ١ = ٢$

عندما $س = ٢ \leftarrow ص = ٢^{٢} + ١ = ٥$

$\int_{٢}^{٥} ٤\, ق(ص)\, دص = ٤ \int_{٢}^{٥} ق(ص)\, دص = ٤ \times -٣ = -١٢$

٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$$\int_{٠}^{٤} ٢س \sqrt{س^{٢} + ٩}\, دس$$

الحل

$\int_{٠}^{٤} ٢س\, (س^{٢} + ٩)^{\frac{١}{٢}} \,.\, دس$

$ص = س^{٢} + ٩ \Leftarrow \frac{دص}{دس} = ٢س \Leftarrow$

$\frac{دص}{٢س} = دس \,.$

$\int_{٠}^{٤} \cancel{٢س} \,.\, ص^{\frac{١}{٢}} \frac{دص}{\cancel{٢س}} = \int_{٠}^{٤} ص^{\frac{١}{٢}} \,.\, دص = \left[\frac{ص^{\frac{١}{٢}+١}}{\frac{١}{٢}+١} \right]_{٠}^{٤} = \frac{٢}{٣} \left[ص^{\frac{٣}{٢}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left[\sqrt{(س^{٢} + ٩)^{٣}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left(\sqrt{(٤^{٢} + ٩)^{٣}} - \sqrt{(٠ + ٩)^{٣}} \right)$

$\frac{٢}{٣} \left(٥^{٣} - ٣^{٣} \right) = \frac{٢}{٣} (١٢٥ - ٢٧)$

$= \frac{٢}{٣} \times ٩٨ = \frac{١٩٦}{٣}$