# إجابات أسئلة الدرس

## التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, د\text{س}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, د\text{س}$

جـ) $\int (2\text{س} - 3\text{س}^2) \, \text{قا}^2(\text{س}^2 - \text{س}^3) \, د\text{س}$

د ) $\int \frac{3\text{س} - 9}{(\text{س}^2 - 6\text{س})^2} \, د\text{س}$

**الحل**

أ) $\int (1-2\text{س})(\text{س}-\text{س}^2)^4 \, د\text{س}$ .

$\text{ص} = \text{س} - \text{س}^2 \Leftarrow \frac{د\text{ص}}{د\text{س}} = 1 - 2\text{س} \Leftarrow \frac{د\text{ص}}{1-2\text{س}} = د\text{س}$

$\int \cancel{(1-2\text{س})} \, \text{ص}^4 \frac{د\text{ص}}{\cancel{1-2\text{س}}} = \int \text{ص}^4 \, د\text{ص}$

$= \frac{\text{ص}^5}{5} + \text{جـ} = \frac{(\text{س}-\text{س}^2)^5}{5} + \text{جـ}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, د\text{س}$

$\text{ص} = 2\text{س}^3 - 2 \Leftarrow \frac{د\text{ص}}{د\text{س}} = 6\text{س}^2 \Leftarrow \frac{د\text{ص}}{6\text{س}^2} = د\text{س}$

$\int \cancel{6\text{س}^2} \sqrt[5]{\text{ص}^2} \frac{د\text{ص}}{\cancel{6\text{س}^2}} = \int \text{ص}^{\frac{2}{5}} \, د\text{ص}$

$$\frac{\text{ص}^{\frac{2}{5}+1}}{\frac{2}{5}+1} + \text{جـ} = \frac{\text{ص}^{\frac{7}{5}}}{\frac{7}{5}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{\text{ص}^{7}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{(2\text{س}^{3}-2)^{7}} + \text{جـ}$$

جـ) $\int (2\text{س} - 3\text{س}^2)\,\text{قا}^2(\text{س}^3 - \text{س}^2)\,\text{دس}$

$$\text{ص} = \text{س}^3 - \text{س}^2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 3\text{س}^2 - 2\text{س}$$

$$\frac{\text{دص}}{3\text{س}^2 - 2\text{س}} = \text{دس}\,.$$

$$\int \overset{-1}{\cancel{(2\text{س} - 3\text{س}^2)}}\,\text{قا}^2\text{ص}\,\frac{\text{دص}}{\cancel{3\text{س}^2 - 2\text{س}}}$$

$$\int -\text{قا}^2\text{ص}\cdot\text{دص} = -\text{ظا}\,\text{ص} + \text{جـ}$$

$$= -\text{ظا}(\text{س}^3 - \text{س}^2) + \text{جـ}$$

د) $\int \frac{3\text{س} - 9}{(\text{س}^2 - 6\text{س})^2}\,\text{دس}$

$$\text{ص} = \text{س}^2 - 6\text{س} \Leftarrow \frac{\text{دص}}{\text{دس}} = 2\text{س} - 6 \Leftarrow$$

$$\frac{\text{دص}}{2\text{س} - 6} = \text{دس}\,.$$

$$\int \frac{3\text{س} - 9}{\text{ص}^2} \times \frac{\text{دص}}{2\text{س} - 6} =$$

$$\int 3\cancel{(\text{س} - 3)} \times \text{ص}^{-2} \times \frac{\text{دص}}{2\cancel{(\text{س} - 3)}} =$$

$$\frac{3}{2}\,\frac{\text{ص}^{-2+1}}{-2+1} + \text{جـ} = \frac{3}{2}\,\frac{\text{ص}^{-1}}{-1} + \text{جـ}$$

$$-\frac{3}{2\text{ص}} + \text{جـ} = -\frac{3}{2(\text{س}^2 - 6\text{س})} + \text{جـ}$$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{ءس}$

ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{ءس}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{ءس}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{ءس}$

**الحل**

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{ءس} = \int (\text{٣س} - ٢)^{\frac{٢}{٣}}\ \text{ءس} = \dfrac{(\text{٣س} - ٢)^{\frac{٢}{٣} + ١}}{(\frac{٢}{٣} + ١) \times ٣} + \text{جـ}$

$= \dfrac{(\text{٣س} - ٢)^{\frac{٥}{٣}}}{\cancel{٣} \times \frac{٥}{\cancel{٣}}} + \text{جـ} = \frac{١}{٥} \sqrt[3]{(\text{٣س} - ٢)^{٥}} + \text{جـ}$

ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{ءس}$

$\text{ص} = \text{٢س}^{٢} - \text{٤س} + ١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س} - ٤$

$\text{ءس} = \dfrac{\text{ءص}}{\text{٤س} - ٤} \Leftarrow \text{ءس} = \dfrac{\text{ءص}}{٤(\text{س} - ١)}$

$\int \cancel{(\text{س} - ١)}\ \text{ص}^{٥} \times \dfrac{\text{ءص}}{٤\cancel{(\text{س} - ١)}} = \frac{١}{٤} \int \text{ص}^{٥}\ \text{ءص}$

$= \dfrac{\text{ص}^{٦}}{٦ \times ٤} + \text{جـ} = \dfrac{(\text{٢س}^{٢} - \text{٤س} + ١)^{٦}}{٢٤} + \text{جـ}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{ءس} =$

$\dfrac{٢\ \text{ظا}(٢ - \text{س})}{-١} + \text{جـ}$

$-٢\ \text{ظا}(٢ - \text{س}) + \text{جـ}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{ءس}$

$\text{ص} = \text{س}^{٤} + ١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س}^{٣} \Leftarrow$

$\dfrac{\text{ءص}}{\text{٤س}^{٣}} = \text{ءس}$

$\int ٢\cancel{\text{س}^{٣}}\ \text{جا ص}\ \dfrac{\text{ءص}}{٤\cancel{\text{س}^{٣}}} =$

$\int \frac{١}{٢}\ \text{جا ص ءص} = -\frac{١}{٢}\ \text{جتا ص} + \text{جـ}$

$= -\frac{١}{٢}\ \text{جتا}(\text{س}^{٤} + ١) + \text{جـ}$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{0}^{2} \sqrt{4\text{س} + 1}\ \text{دس}$

ب) $\int_{-1}^{-1} 3\text{س}^2(\text{س}^3 - 1)^3\ \text{دس}$

جـ) $\int_{1}^{2} 2\text{س}\ \sqrt[3]{\text{س}^2 - 1}\ \text{دس}$

د) $\int_{1}^{2} \frac{3 - 2\text{س}}{(\text{س}^2 - 3\text{س})^2}\ \text{دس}$

**الحل**

أ) $\int_{0}^{2} \sqrt{4\text{س}+1}\ \text{دس} = \int_{0}^{2} (4\text{س}+1)^{\frac{1}{2}} \cdot \text{دس}$

$$= \left[ \frac{(4\text{س}+1)^{\frac{1}{2}+1}}{4 \times (\frac{1}{2}+1)} \right]_{0}^{2} = \left[ \frac{(4\text{س}+1)^{\frac{3}{2}}}{4 \times \frac{3}{2}} \right]_{0}^{2}$$

$$\left[ \frac{\sqrt{(4\text{س}+1)^3}}{6} \right]_{0}^{2}$$

$$\frac{1}{6}\left[ \sqrt{(4 \times 2 + 1)^3} - \sqrt{(4 \times 0 + 1)^3} \right]$$

$$\frac{1}{6}\left( \sqrt{9^3} - \sqrt{1^3} \right)$$

$$\frac{1}{6}\left(27-1\right)$$

$$\frac{1}{6}\times 26 = \frac{13}{3}$$

ب) $\int_{-1}^{1} 3\text{س}^2\left(\text{س}^3-1\right)^3 \, d\text{س} = \text{صفر}$

جـ) $\int_{1}^{0} 2\text{س}\sqrt[3]{\text{س}^2-1}\;.\;d\text{س} =$

$$\int_{1}^{0} 2\text{س}\left(\text{س}^2-1\right)^{\frac{1}{3}} d\text{س}$$

$$\text{ص} = \text{س}^2-1 \Leftarrow \frac{d\text{ص}}{d\text{س}} = 2\text{س} \Leftarrow d\text{س} = \frac{d\text{ص}}{2\text{س}}$$

$$\int_{1}^{0} \cancel{2\text{س}}\,\text{ص}^{\frac{1}{3}}\frac{d\text{ص}}{\cancel{2\text{س}}} = \int_{1}^{0}\text{ص}^{\frac{1}{3}}\,d\text{ص}$$

$$\left[\frac{\text{ص}^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{1}^{0} = \left[\frac{\text{ص}^{\frac{4}{3}}}{\frac{4}{3}}\right]_{1}^{0}$$

$$\frac{3}{4}\left[\sqrt[3]{\left(\text{س}^2-1\right)^4}\right]_{1}^{0} = \frac{3}{4}\left(\sqrt[3]{\left(0^2-1\right)^4} - \sqrt[3]{\left(1-1\right)^4}\right)$$

$$\frac{3}{4}\left(\sqrt[3]{1} - \sqrt[3]{0}\right)$$

$$\frac{3}{4}\left(1 - 0\right)$$

$$\frac{3}{4} \times 1 = \frac{3}{4}$$

د) $$\int_{1}^{2} \frac{2س - 3}{(س^2 - 3س)^2}\, دس = \int_{1}^{2} (2س - 3)(س^2 - 3س)^{-2} \cdot دس$$

$$ص = س^2 - 3س \Leftarrow \frac{دص}{دس} = 2س - 3 \Leftarrow دس = \frac{دص}{2س - 3}$$

$$\int_{1}^{2} \cancel{(2س - 3)}\, ص^{-2} \times \frac{دص}{\cancel{2س - 3}} = \int_{1}^{2} ص^{-2}\, دص =$$

(-1)

$$\left[\frac{-ص^{-2+1}}{-2+1}\right]_{1}^{2} = \left[\frac{-ص^{-1}}{-1}\right]_{1}^{2} = \left[\frac{1}{ص}\right]_{1}^{2}$$

$$\left[\frac{1}{س^2 - 3س}\right]_{1}^{2} = \frac{1}{2^2 - 3 \times 2} - \frac{1}{1^2 - 3 \times 1} = \frac{1}{-2} - \frac{1}{-2}$$

$$= \frac{-1}{2} + \frac{1}{2} = \text{صفر}$$

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-2}^{3} 3س^2\, قَ(س^3)\, دس$

**الحل**

$$ص = س^3 \Leftarrow \frac{دص}{دس} = 3س^2 \Leftarrow دس = \frac{دص}{3س^2}$$

$$\int_{-2}^{3} \cancel{3س^2}\, قَ(ص) \frac{دص}{\cancel{3س^2}} = \int_{-2}^{3} قَ(ص) \cdot دص =$$

$$\left[ق(ص)\right]_{-2}^{3} = \left[ق(س^3)\right]_{-2}^{3}$$

$$ق(3^3) - ق((-2)^3) = ق(27) - ق(-8) = -6 - 5$$

$$= -11$$

٥) إذا علمت أن $\int_{٥}^{٢} ق(س)\, دس = ٣$، فجد قيمة التكامل الآتي: $\int_{-١}^{٢} ٨س\, ق(س^{٢} + ١)\, دس$

**الحل**

$ص = س^{٢} + ١ \Leftarrow \frac{دص}{دس} = ٢س \Leftarrow \frac{دص}{٢س} = دس$

$\int_{-١}^{٢} ٨\cancel{س}\, ق(ص) \frac{دص}{٢\cancel{س}} = ٤\int_{-١}^{٢} ق(ص)\, دص$

عندما $س = -١ \leftarrow ص = (-١)^{٢} + ١ = ٢$

عندما $س = ٢ \leftarrow ص = ٢^{٢} + ١ = ٥$

$٤\int_{٢}^{٥} ق(ص)\, دص = ٤\int_{٢}^{٥} ق(ص)\, دص = ٤ \times -٣ = -١٢$

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٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$$\int_{٠}^{٤} ٢س \sqrt{س^{٢} + ٩}\, دس$$

**الحل**

$\int_{٠}^{٤} ٢س\, (س^{٢} + ٩)^{\frac{١}{٢}} .\, دس$

$ص = س^{٢} + ٩ \Leftarrow \frac{دص}{دس} = ٢س \Leftarrow$

$\frac{دص}{٢س} = دس$ .

$\int_{٠}^{٤} \cancel{٢س} .\, ص^{\frac{١}{٢}} \frac{دص}{\cancel{٢س}} = \int_{٠}^{٤} ص^{\frac{١}{٢}} .\, دص = \left[ \frac{ص^{\frac{١}{٢}+١}}{\frac{١}{٢}+١} \right]_{٠}^{٤} = \left[ \frac{٢}{٣} ص^{\frac{٣}{٢}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left[ \sqrt{(س^{٢} + ٩)^{٣}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left( \sqrt{(٤^{٢} + ٩)^{٣}} - \sqrt{(٠ + ٩)^{٣}} \right)$

$\frac{٢}{٣} \left( ٥^{٣} - ٣^{٣} \right) = \frac{٢}{٣} (١٢٥ - ٢٧)$

$= \frac{٢}{٣} \times ٩٨ = \frac{١٩٦}{٣}$