

إجابات كتاب التمارين

التكامل المحدود

أجد قيمة كل من التكاملات الآتية:

$$\int (1510x - 2) dx$$

$$\int (1510x - 2) dx = \left(\frac{1510x^2}{2} - 2x \right) \Big|_{15} = \left(\frac{1510 \times 15^2}{2} - 2 \times 15 \right) - \left(\frac{1510 \times 1^2}{2} - 2 \times 1 \right) = 87$$

$$\int (2x^3 - 4x + 5) dx$$

$$\int (2x^3 - 4x + 5) dx = \left(\frac{2x^4}{4} - 2x^2 + 5x \right) \Big|_{02} = \left(\frac{2 \times 8}{4} - 2 \times 4 + 5 \times 2 \right) - \left(\frac{2 \times 0}{4} - 2 \times 0 + 5 \times 0 \right) = 10$$

$$\int (14x^3 + 2x^2) dx$$

$$\int (14x^3 + 2x^2) dx = \left(\frac{14x^4}{4} + \frac{2x^3}{3} \right) \Big|_{52} = \left(\frac{14 \times 5^4}{4} + \frac{2 \times 5^3}{3} \right) - \left(\frac{14 \times 2^4}{4} + \frac{2 \times 2^3}{3} \right) = \left(\frac{14 \times 625}{4} + \frac{2 \times 125}{3} \right) - \left(\frac{14 \times 16}{4} + \frac{2 \times 8}{3} \right) = 2138 \frac{5}{12}$$

$$\int (x - 3x^2) dx$$

$$\int (x - 3x^2) dx = \left(\frac{x^2}{2} - x^3 \right) \Big|_{36} = \left(\frac{3^2}{2} - 3^3 \right) - \left(\frac{6^2}{2} - 6^3 \right) = \left(\frac{9}{2} - 27 \right) - \left(\frac{36}{2} - 216 \right) = 93$$

$$\int (x + 3) |x - 5| dx$$

$$\int (x + 3) |x - 5| dx = \int_{-3}^{-5} (x + 3) (-x + 5) dx + \int_{-3}^{\infty} (x + 3) (x - 5) dx = \int_{-3}^{-5} (-x^2 + 2x + 15) dx + \int_{-3}^{\infty} (x^2 - 2x - 15) dx = \left(-\frac{x^3}{3} + x^2 + 15x \right) \Big|_{-3}^{-5} + \left(\frac{x^3}{3} - x^2 - 15x \right) \Big|_{-3}^{\infty} = \left(-\frac{125}{3} + 25 - 75 \right) - \left(-\frac{27}{3} + 9 - 45 \right) + \left(\frac{x^3}{3} - x^2 - 15x \right) \Big|_{-3}^{\infty} = -\frac{100}{3} - \left(\frac{27}{3} - 9 - 45 \right) = -\frac{100}{3} - \left(9 - 45 \right) = -\frac{100}{3} + 36 = -\frac{28}{3}$$

$$\int (6x(6 - x)) dx$$

$$\int (6x(6 - x)) dx = \int (36x - 6x^2) dx = \left(\frac{36x^2}{2} - \frac{6x^3}{3} \right) \Big|_{06} = \left(18x^2 - 2x^3 \right) \Big|_{06} = \left(18 \times 6^2 - 2 \times 6^3 \right) - \left(18 \times 0^2 - 2 \times 0^3 \right) = 108 - 72 = 36$$

$$\int (6x - 12x^4 + 3) dx$$

$$\int (6x - 12x^4 + 3) dx = \left(\frac{6x^2}{2} - \frac{12x^5}{5} + 3x \right) \Big|_{12} = \left(3x^2 - \frac{12x^5}{5} + 3x \right) \Big|_{12} = \left(3 \times 12^2 - \frac{12 \times 12^5}{5} + 3 \times 12 \right) - \left(3 \times 1^2 - \frac{12 \times 1^5}{5} + 3 \times 1 \right) = \left(432 - \frac{12 \times 248832}{5} + 36 \right) - \left(3 - \frac{12}{5} + 3 \right) = 468 - \frac{298608}{5} + \frac{12}{5} = 468 - 59721.6 + 2.4 = -59251.2$$

$$\int_{-2}^2 (2x-1) dx \quad (8) \quad 07 \int$$

$$\int_{-2}^2 (2x-1) dx = \int_{-2}^0 (-2x+1) dx + \int_0^2 (2x-1) dx = \left[-x^2 + x \right]_{-2}^0 + \left[x^2 - x \right]_0^2 = (-14 + 12) - (0) + (49 - 7) - (0 - 12) = 852$$

$$\int_{-3}^4 |x| dx \quad (9) \quad 34 - \int$$

$$\int_{-3}^4 |x| dx = \int_{-3}^0 -x dx + \int_0^4 x dx = -\left[\frac{1}{2}x^2 \right]_{-3}^0 + \left[\frac{1}{2}x^2 \right]_0^4 = (0) - (-92) + (8) - (0) = 252$$

$$\int_{-2}^2 (12x^2 + x^3) dx \quad (10) \int$$

$$\int_{-2}^2 (12x^2 + x^3) dx = \int_{-2}^2 12(x^2 + x^3) dx = \int_{-2}^2 12(x + x^2) dx = \left[12x^2 + 13x^3 \right]_{-2}^2 = (2 \int + 83) - (12 + 13) = 32 + 73 = 236$$

$$\int_{-4}^6 (6x^2 - 4x) dx \quad (11) \quad 34 \int$$

$$\int_{-4}^6 (6x^2 - 4x) dx = \left[2x^3 + 2x^2 \right]_{-4}^6 = (128 + 32) - (54 + 18) = 88 \quad 34 \int$$

$$\int_{-1}^1 (10x + 1x^2) dx \quad (12) \int$$

$$\int_{-1}^1 (10x + 1x^2) dx = 0 \int$$

إذا كان $\int_{-2}^6 g(x) dx = -2$ ، $\int_{-1}^2 f(x) dx = 4$ ، $\int_{-3}^2 f(x) dx = 5$ ، فأجد كلاً مما يأتي:

$$\int_{-2}^2 2f(x) dx \quad (13) \int$$

$$\int_{-2}^2 2f(x) dx = 0 \int$$

$$\int_{-2}^2 (f(x) - 5) dx \quad (14) \quad 12 \int$$

$$\int_{-2}^2 (f(x) - 5) dx = \int_{-2}^2 f(x) dx - \int_{-2}^2 5 dx = \int_{-2}^2 (1 - 3f(x)) dx + \int_{-2}^2 (-32f(x) + 12 - 5) dx = -4 + 5 + (-5x) \Big|_{-2}^2 = 1 + (-10) - (-5) = -4$$

$$\int_{-2}^2 (2f(x) + 5g(x)) dx \quad (15) \quad 32 - \int$$

$$\int_{-2}^2 (2f(x) + 5g(x)) dx = -2 \int_{-2}^2 f(x) dx + 5 \int_{-2}^2 g(x) dx = -2(5) + 5(-2) = -32 - \int$$

$$=-20$$

$$\int (g(x)+2x)dx \quad (16) \quad 2-3 \int$$

$$\int (g(x)+2x)dx = \int 2-3g(x)dx + \int 2-32xdx = -(-2) + (x^2)|_{2-3} = 2+9-(-)2-3 \int$$

$$4=7$$

$$\int (f(x)+g(x))dx \quad (17) \quad 2-3 \int$$

$$\int (f(x)+g(x))dx = \int 2-3f(x)dx + \int 2-3g(x)dx = -5+2 = -3)2-3 \int$$

$$\int (4f(x)-3g(x))dx \quad (18) \quad 32-\int$$

$$\int (4f(x)-3g(x))dx = 4\int -32f(x)dx - 3\int -32g(x)dx = 4(5) - 3(-2) = 26)32-\int$$

(19) إذا كان $f(x) = \begin{cases} x^2, & x < 2 \\ 28-x, & x \geq 2 \end{cases}$ ، فأجد قيمة $\int -36f(x)dx$.

$$\int -36f(x)dx = \int -32f(x)dx + \int 26f(x)dx = \int -32x^2dx + \int 26(8-x)dx = (13x^3) - \int$$

$$|-32 + (8x - 12x^2)|_{26} = (83) - (-9) + (48 - 18) - (16 - 2) = 833$$

(20) سكان: أشارت دراسة إلى أن عدد السكان في إحدى القرى يتغير شهرياً بمعدل يمكن نمذجته بالاقتران: $P'(t) = 5 + 3t^{2/3}$ ، حيث t عدد الأشهر من الآن، و $P(t)$ عدد السكان، أجد مقدار الزيادة في عدد سكان القرية في الأشهر الثمانية القادمة.

$$P(t) = \int_0^8 (5 + 3t^{2/3})dt = (5t + 95t^{5/3})|_0^8 = (40 + 2885) - (0) = 4885$$

(21) إذا كان: $\int (x^2 - a)dx = 5)23 \int$ ، فأجد قيمة الثابت a .

$$\int (x^2 - a)dx = 5(13x^3 - ax)|_{23} = 5(9 - 3a) - (83 - 2a) = 5173 - \alpha = 5a = 23)23 \int$$