

أدرب وأحل المسائل

التكامل المحدود

أجد قيمة كل من التكاملات الآتية:

(1) $\int -133x^2 dx$

$$\int -133x^2 dx = x^3|_{-13} = (3)^3 - (-1)^3 = 28$$

(2) $\int -3-26 dx$

$$\int -3-26 dx = 6x|_{-3-2} = 6(-2) - 6(-3) = 6$$

(3) $\int_0^2 (3x^2+4x+3) dx$

$$\int_0^2 (3x^2+4x+3) dx = (x^3+2x^2+3x)|_0^2 = ((2)^3+2(2)^2+3(2)) - ((0)^3+2(0)^2+3(0)) = 22$$

(4) $\int 188x^3 dx$

$$\int 188x^3 dx = \int 188x^13 dx = 6x^43|_{18} = 6x^43|_{18} = 6843 - 6043 = 96$$

(5) $\int 19(x-4x) dx$

$$\int 19(x-4x) dx = \int 19(x^{12}-4x^{-12}) dx = (23x^{32}-8x^{12})|_{19} = (23x^3-8x)|_{19} = (239^3-89) - (231^3-81) = 43$$

(6) $\int -23(-x^2+4x-5) dx$

$$\int -23(-x^2+4x-5) dx = (-13x^3+2x^2-5x)|_{-23} = (-13(3)^3+2(3)^2-5(3)) - (-13(-2)^3+2(-2)^2-5(-2)) = -803$$

(7) $\int 13(x-2)(x+2) dx$

$$\int 13(x-2)(x+2) dx = \int 13(x^2-4) dx = (13x^3-4x)|_{13} = (13(3)^3-4(3)) - (13(1)^3-4(1)) = 23$$

(8) $\int -33(9-x^2) dx$

$$\int -33(9-x^2) dx = (9x - 13x^3) \Big|_{-33} = (9(3) - 13(3)^3) - (9(-3) - 13(-3)^3) = 36$$

(9) $\int 14x^2 + x dx$

$$\int 14x^2 + x dx = \int 14(2x^2 + xx^2) dx = \int 14(2x - 2 + x - 32) dx = (-2x - 1 - 2x - 12) \Big|_{14} = (-2x - 2x) \Big|_{14} = (-24 - 24) - (-21 - 21) = 52$$

(10) $\int 14x^3(x+1x) dx$

$$\int 14x^3(x+1x) dx = \int 14x^3(x^2+x-1) dx = \int 14(x^7+x^2) dx = (29x^9 + 13x^3) \Big|_{14} = (29x^9 + 13x^3) \Big|_{14} = (2949 + 13(4)^3) - (2919 + 13(1)^3) = 12119$$

(11) $\int 18(x^{1/3} - x - 1/5) dx$

$$\int 18(x^{1/3} + x - 15) dx = (34x^{4/3} + 54x^45) \Big|_{18} = (34x^{4/3} + 54x^45) \Big|_{18} = (34843 + 54845) - (34143 + 54145) = 10 + 54845$$

(12) $\int 19(2+x)^2 dx$

$$\int 19(2+x)^2 dx = \int 19(4+4x+x^2) dx = \int 19(4+4x^2+x) dx = (4x+83x^3 + 12x^2) \Big|_{19} = (4(9)+83(9)^3 + 12(9)^2) - (4(1)+83(1)^3 + 12(1)^2) = 4243$$

(13) $\int -14|3x-6| dx$

أعيد تعريف اقتران القيمة المطلقة:

$$|3x-6| = \begin{cases} 6-3x, & x < 2 \\ 3x-6, & x \geq 2 \end{cases}$$

بما أن الاقتران تشعب عند 2 ؛ فإنني أجزئ التكامل عنده:

$$\int -14|3x-6| dx = \int -12(6-3x) dx + \int 24(3x-6) dx = (6x-32x^2) \Big|_{-12} + (32x^2-6x) \Big|_{24} = (6(2)-32(2)^2) - (6(-1)-32(-1)^2) + (32(4)^2-6(4)) - (32(2)^2-6(2)) = 392$$

(14) $\int 03|x-2| dx$

أعيد تعريف اقتران القيمة المطلقة:

$$|x-2| = \begin{cases} 2-x, & x < 2 \\ x-2, & x \geq 2 \end{cases}$$

بما أن الاقتران تشعب عند 2 ؛ فإنني أجزئ التكامل عنده:

$$\int_0^3 |x-2| dx = \int_0^2 (2-x) dx + \int_2^3 (x-2) dx = (2x - \frac{1}{2}x^2) \Big|_0^2 + (\frac{1}{2}x^2 - 2x) \Big|_2^3 = (2(2) - \frac{1}{2}(2)^2) - (2(0) - \frac{1}{2}(0)^2) + (\frac{1}{2}(3)^2 - 2(3)) - (\frac{1}{2}(2)^2 - 2(2)) = 5/2$$

(15) $\int_2^3 x^2 - 1x + 1 dx$

$$\int_2^3 x^2 - 1x + 1 dx = \int_2^3 (x+1)(x-1)x + 1 dx = \int_2^3 (x-1) dx = (\frac{1}{2}x^2 - x) \Big|_2^3 = (\frac{1}{2}(3)^2 - 3) - (\frac{1}{2}(2)^2 - 2) = 3/2$$