إجابات أسئلة الدرس

التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ) $\int (\text{١} - \text{٢}\text{س})(\text{س} - \text{س}^{\text{٢}})^{\text{٤}}\, \text{دس}$

ب) $\int \text{٦}\text{س}^{\text{٢}} \sqrt[\text{٥}]{(\text{٢}\text{س}^{\text{٣}} - \text{٢})^{\text{٢}}}\, \text{دس}$

جـ) $\int (\text{٢}\text{س} - \text{٣}\text{س}^{\text{٢}})\, \text{قا}^{\text{٢}}(\text{س}^{\text{٢}} - \text{س}^{\text{٣}})\, \text{دس}$

د) $\int \frac{\text{٣}\text{س} - \text{٩}}{(\text{س}^{\text{٢}} - \text{٦}\text{س})^{\text{٢}}}\, \text{دس}$

الحل

أ) $\int (\text{١} - \text{٢}\text{س})(\text{س} - \text{س}^{\text{٢}})^{\text{٤}}\, \text{دس}$.

$\text{ص} = \text{س} - \text{س}^{\text{٢}} \Leftarrow \frac{\text{دص}}{\text{دس}} = \text{١} - \text{٢}\text{س} \Leftarrow \frac{\text{دص}}{\text{١} - \text{٢}\text{س}} = \text{دس}$

$\int \cancel{(\text{١} - \text{٢}\text{س})}\, \text{ص}^{\text{٤}} \frac{\text{دص}}{\cancel{\text{١} - \text{٢}\text{س}}} = \int \text{ص}^{\text{٤}}\, \text{دص}$

$= \frac{\text{ص}^{\text{٥}}}{\text{٥}} + \text{جـ} = \frac{(\text{س} - \text{س}^{\text{٢}})^{\text{٥}}}{\text{٥}} + \text{جـ}$

ب) $\int \text{٦}\text{س}^{\text{٢}} \sqrt[\text{٥}]{(\text{٢}\text{س}^{\text{٣}} - \text{٢})^{\text{٢}}}\, \text{دس}$

$\text{ص} = \text{٢}\text{س}^{\text{٣}} - \text{٢} \Leftarrow \frac{\text{دص}}{\text{دس}} = \text{٦}\text{س}^{\text{٢}} \Leftarrow \frac{\text{دص}}{\text{٦}\text{س}^{\text{٢}}} = \text{دس}$

$\int \cancel{\text{٦}\text{س}^{\text{٢}}} \sqrt[\text{٥}]{\text{ص}^{\text{٢}}} \frac{\text{دص}}{\cancel{\text{٦}\text{س}^{\text{٢}}}} = \int \text{ص}^{\frac{\text{٢}}{\text{٥}}}\, \text{دص}$

$$\frac{\text{ص}^{\frac{2}{5}+1}}{\frac{2}{5}+1} + \text{جـ} = \frac{\text{ص}^{\frac{7}{5}}}{\frac{7}{5}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{\text{ص}^{7}} + \text{جـ}$$

$$= \frac{5}{7}\sqrt[5]{(2\text{س}^{3} - 2)^{7}} + \text{جـ}$$

جـ) $\int (2\text{س} - 3\text{س}^{2})\,\text{قا}^{2}(\text{س}^{2} - \text{س}^{3})\,\text{دس}$

$\text{ص} = \text{س}^{2} - \text{س}^{3} \Leftarrow \frac{\text{دص}}{\text{دس}} = 2\text{س} - 3\text{س}^{2}$

$\frac{\text{دص}}{2\text{س} - 3\text{س}^{2}} = \text{دس}$.

$$\int \overset{-1}{\cancel{(2\text{س} - 3\text{س}^{2})}}\,\text{قا}^{2}\text{ص}\,\frac{\text{دص}}{\cancel{3\text{س}^{2} - 2\text{س}}}$$

$$\int -\text{قا}^{2}\text{ص}\cdot\text{دص} = -\text{ظا ص} + \text{جـ}$$

$$= -\text{ظا}(\text{س}^{2} - \text{س}^{3}) + \text{جـ}$$

د) $\int \frac{3\text{س} - 9}{(\text{س}^{2} - 6\text{س})^{2}}\,\text{دس}$

$\text{ص} = \text{س}^{2} - 6\text{س} \Leftarrow \frac{\text{دص}}{\text{دس}} = 2\text{س} - 6 \Leftarrow$

$\frac{\text{دص}}{2\text{س} - 6} = \text{دس}$.

$$\int \frac{3\text{س} - 9}{\text{ص}^{2}} \times \frac{\text{دص}}{2\text{س} - 6} =$$

$$\int 3\cancel{(\text{س} - 3)} \times \text{ص}^{-2} \times \frac{\text{دص}}{2\cancel{(\text{س} - 3)}} =$$

$$\frac{3}{2}\,\frac{\text{ص}^{-2+1}}{-2+1} + \text{جـ} = \frac{3}{2}\,\frac{\text{ص}^{-1}}{-1} + \text{جـ}$$

$$\frac{-3}{2\text{ص}} + \text{جـ} = \frac{-3}{2(\text{س}^{2} - 6\text{س})} + \text{جـ}$$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{دس}$

ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{دس}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{دس}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{دس}$

الحل

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{دس} = \int (\text{٣س} - ٢)^{\frac{٢}{٣}}\ \text{دس} = \dfrac{(\text{٣س} - ٢)^{\frac{٢}{٣} + ١}}{(\frac{٢}{٣} + ١) \times ٣} + \text{جـ}$

$= \dfrac{(\text{٣س} - ٢)^{\frac{٥}{٣}}}{\cancel{٣} \times \frac{٥}{\cancel{٣}}} + \text{جـ} = \dfrac{١}{٥} \sqrt[3]{(\text{٣س} - ٢)^{٥}} + \text{جـ}$

ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{دس}$

$\text{ص} = \text{٢س}^{٢} - \text{٤س} + ١ \Leftarrow \dfrac{\text{دص}}{\text{دس}} = \text{٤س} - ٤$

$\dfrac{\text{دص}}{\text{٤س} - ٤} = \text{دس} \Leftarrow \dfrac{\text{دص}}{٤(\text{س} - ١)} = \text{دس}$

$\int \cancel{(\text{س} - ١)}\ \text{ص}^{٥} \times \dfrac{\text{دص}}{٤\cancel{(\text{س} - ١)}} = \dfrac{١}{٤} \int \text{ص}^{٥}\ \text{دص}$

$= \dfrac{\text{ص}^{٦}}{٦ \times ٤} + \text{جـ} = \dfrac{(\text{٢س}^{٢} - \text{٤س} + ١)^{٦}}{٢٤} + \text{جـ}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{دس} =$

$\dfrac{٢\ \text{ظا}(٢ - \text{س})}{-١} + \text{جـ}$

$-٢\ \text{ظا}(٢ - \text{س}) + \text{جـ}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{دس}$

$\text{ص} = \text{س}^{٤} + ١ \Leftarrow \dfrac{\text{دص}}{\text{دس}} = \text{٤س}^{٣} \Leftarrow$

$\dfrac{\text{دص}}{\text{٤س}^{٣}} = \text{دس}$

$\int ٢\cancel{\text{س}^{٣}}\ \text{جا ص}\ \dfrac{\text{دص}}{٤\cancel{\text{س}^{٣}}} =$

$\int \dfrac{١}{٢}\ \text{جا ص}\ \text{دص} = -\dfrac{١}{٢}\ \text{جتا ص} + \text{جـ}$

$= -\dfrac{١}{٢}\ \text{جتا}(\text{س}^{٤} + ١) + \text{جـ}$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{0}^{2} \sqrt{4س + 1}\ دس$

ب) $\int_{-1}^{-1} 3س^2(س^3 - 1)^3\ دس$

جـ) $\int_{1}^{0} 2س\sqrt[3]{س^2 - 1}\ دس$

د) $\int_{1}^{2} \frac{3 - 2س}{(س^2 - 3س)^2}\ دس$

الحل

أ) $\int_{0}^{2} \sqrt{4س+1}\ دس = \int_{0}^{2} (4س+1)^{\frac{1}{2}} . دس$

$$= \left[\frac{(4س+1)^{\frac{1}{2}+1}}{4 \times (\frac{1}{2}+1)}\right]_{0}^{2} = \left[\frac{(4س+1)^{\frac{3}{2}}}{4 \times \frac{3}{2}}\right]_{0}^{2}$$

$$\left[\frac{\sqrt{(4س+1)^3}}{6}\right]_{0}^{2}$$

$$\frac{1}{6}\left[\sqrt{(4 \times 2 + 1)^3} - \sqrt{(4 \times 0 + 1)^3}\right]$$

$$\frac{1}{6}\left(\sqrt{9^3} - \sqrt{1^3}\right)$$

$\frac{1}{6}(27-1)$

$\frac{1}{6} \times 26 = \frac{13}{3}$

ب) $\int_{-1}^{1} 3س^2 (س^3-1)^3 \, دس = صفر$

جـ) $\int_{1}^{0} 2س \sqrt[3]{س^2-1} \, . \, دس =$

$\int_{1}^{0} 2س (س^2-1)^{\frac{1}{3}} \, دس$

$ص = س^2 - 1 \Leftarrow \frac{دص}{دس} = 2س \Leftarrow دس = \frac{دص}{2س}$

$\int_{1}^{0} \cancel{2س} \, ص^{\frac{1}{3}} \frac{دص}{\cancel{2س}} = \int_{1}^{0} ص^{\frac{1}{3}} \, دص$

$\left[\frac{ص^{\frac{1}{3}+1}}{\frac{1}{3}+1} \right]_{1}^{0} = \left[\frac{ص^{\frac{4}{3}}}{\frac{4}{3}} \right]_{1}^{0}$

$\frac{3}{4} \left[\sqrt[3]{(س^2-1)^4} \right]_{1}^{0} = \frac{3}{4} \left(\sqrt[3]{(0^2-1)^4} - \sqrt[3]{(1-1)^4} \right)$

$$\frac{٣}{٤}\left(\sqrt[٢]{١} - \sqrt[٣]{٠}\right)$$

$$\frac{٣}{٤}\left(١ - ٠\right)$$

$$\frac{٣}{٤} \times ١ = \frac{٣}{٤}$$

ج) $\int_{١}^{٢} \frac{٢س - ٣}{(س^٢ - ٣س)^٢} \, دس = \int_{١}^{٢} (٢س - ٣)(س^٢ - ٣س)^{-٢} \, . \, دس$

$ص = س^٢ - ٣س \Leftarrow \frac{دص}{دس} = ٢س - ٣ \Leftarrow دس = \frac{دص}{٢س - ٣}$

$\int_{١}^{٢} \cancel{(٢س - ٣)} \, ص^{-٢} \, \frac{دص}{\cancel{٢س - ٣}} = \int_{١}^{٢} ص^{-٢} \, دص =$ (١-)

$\left[\frac{ص^{-٢+١}}{-٢+١}\right]_{١}^{٢} = \left[\frac{ص^{-١}}{-١}\right]_{١}^{٢} = \left[\frac{١}{ص}\right]_{١}^{٢}$

$\left[\frac{١}{س^٢ - ٣س}\right]_{١}^{٢} = \frac{١}{٢^٢ - ٣ \times ٢} - \frac{١}{١^٢ - ٣ \times ١} = \frac{١}{-٢} - \frac{١}{-٢}$

$= \frac{-١}{٢} + \frac{١}{٢} = صفر$

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-٢}^{٣} ٣س^٢ \, قَ(س^٣) \, دس$

الحل

$ص = س^٣ \Leftarrow \frac{دص}{دس} = ٣س^٢ \Leftarrow دس = \frac{دص}{٣س^٢}$

$\int_{-٢}^{٣} \cancel{٣س^٢} \, دَ(ص) \, \frac{دص}{\cancel{٣س^٢}} = \int_{-٢}^{٣} دَ(ص) \, . \, دص =$

$\left[د(ص)\right]_{-٢}^{٣} = \left[د(س^٣)\right]_{-٢}^{٣}$

$د(٣^٣) - د((-٢)^٣) = د(٢٧) - د(-٨) = -٦ - ٥$

$= -١١$

٥) إذا علمت أن $\int_{٥}^{٢}$ ق(س) دس = ٣، فجد قيمة التكامل الآتي: $\int_{-١}^{٢}$ ٨س ق(س^{٢} + ١) دس

الحل

$\text{ص} = \text{س}^{٢} + ١ \Leftarrow \frac{\text{دص}}{\text{دس}} = ٢\text{س} \Leftarrow \frac{\text{دص}}{٢\text{س}} = \text{دس}$

$\int_{-١}^{٢} ٨\text{س}\ \text{ق}(\text{ص})\ \frac{\text{دص}}{٢\text{س}} = \int_{-١}^{٢} ٤\ \text{ق}(\text{ص})\ \text{دص}$

عندما س = -١ ← ص = $(-١)^{٢} + ١ = ٢$

عندما س = ٢ ← ص = $٢^{٢} + ١ = ٥$

$\int_{٢}^{٥} ٤\ \text{ق}(\text{ص})\ \text{دص} = ٤ \int_{٢}^{٥} \text{ق}(\text{ص})\ \text{دص} = ٤ \times -٣ = -١٢$

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٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$\int_{٠}^{٤} ٢\text{س} \sqrt{\text{س}^{٢} + ٩}\ \text{دس}$

الحل

$\int_{٠}^{٤} ٢\text{س}\ (\text{س}^{٢} + ٩)^{\frac{١}{٢}} .\ \text{دس}$

$\text{ص} = \text{س}^{٢} + ٩ \Leftarrow \frac{\text{دص}}{\text{دس}} = ٢\text{س} \Leftarrow$

$\frac{\text{دص}}{٢\text{س}} = \text{دس}$.

$\int_{٠}^{٤} \cancel{٢\text{س}} .\ \text{ص}^{\frac{١}{٢}}\ \frac{\text{دص}}{\cancel{٢\text{س}}} = \int_{٠}^{٤} \text{ص}^{\frac{١}{٢}} .\ \text{دص} = \left[\frac{\text{ص}^{\frac{١}{٢}+١}}{\frac{١}{٢}+١} \right]_{٠}^{٤} = \frac{٢}{٣} \left[\text{ص}^{\frac{٣}{٢}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left[\sqrt{(\text{س}^{٢} + ٩)^{٣}} \right]_{٠}^{٤}$

$\frac{٢}{٣} \left(\sqrt{(٤^{٢} + ٩)^{٣}} - \sqrt{(٠ + ٩)^{٣}} \right)$

$\frac{٢}{٣} \left(٥^{٣} - ٣^{٣} \right) = \frac{٢}{٣} (١٢٥ - ٢٧)$

$= \frac{٢}{٣} \times ٩٨ = \frac{١٩٦}{٣}$