إجابات أسئلة الدرس

التكامل بالتعويض

١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{ds}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{ds}$

جـ) $\int (2\text{س} - 3\text{س}^2) \text{قا}^2(\text{س}^2 - \text{س}^3) \, \text{ds}$

د) $\int \frac{3\text{س} - 9}{(\text{س}^2 - 6\text{س})^2} \, \text{ds}$

الحل

أ) $\int (1 - 2\text{س})(\text{س} - \text{س}^2)^4 \, \text{ds}$.

$\text{ص} = \text{س} - \text{س}^2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 1 - 2\text{س} \Leftarrow \frac{\text{دص}}{1 - 2\text{س}} = \text{دس}$

$\int \cancel{(1 - 2\text{س})} \, \text{ص}^4 \frac{\text{دص}}{\cancel{1 - 2\text{س}}} = \int \text{ص}^4 \, \text{دص}$

$= \frac{\text{ص}^5}{5} + \text{جـ} = \frac{(\text{س} - \text{س}^2)^5}{5} + \text{جـ}$

ب) $\int 6\text{س}^2 \sqrt[5]{(2\text{س}^3 - 2)^2} \, \text{دس}$

$\text{ص} = 2\text{س}^3 - 2 \Leftarrow \frac{\text{دص}}{\text{دس}} = 6\text{س}^2 \Leftarrow \frac{\text{دص}}{6\text{س}^2} = \text{دس}$

$\int \cancel{6\text{س}^2} \sqrt[5]{\text{ص}^2} \frac{\text{دص}}{\cancel{6\text{س}^2}} = \int \text{ص}^{\frac{2}{5}} \, \text{دص}$

$$= \frac{ص^{\frac{2}{5}+1}}{\frac{2}{5}+1} + جـ = \frac{ص^{\frac{7}{5}}}{\frac{7}{5}} + جـ$$

$$= \frac{5}{7}\sqrt[5]{ص^{7}} + جـ$$

$$= \frac{5}{7}\sqrt[5]{(2س^{3}-2)^{7}} + جـ$$

جـ) $\int (2س - 3س^{2})\, قا^{2}(س^{3} - س^{2})\, دس$

$$ص = س^{3} - س^{2} \Leftarrow \frac{دص}{دس} = 3س^{2} - 2س$$

$$\frac{دص}{3س^{2} - 2س} = دس \text{ .}$$

$$\int \cancel{(2س - 3س^{2})}^{-1}\, قا^{2}ص\, \frac{دص}{\cancel{3س^{2} - 2س}}$$

$$\int - قا^{2}ص \cdot دص = - ظاص + جـ$$

$$= - ظا(س^{3} - س^{2}) + جـ$$

د) $\int \frac{3س - 9}{(س^{2} - 6س)^{2}}\, دس$

$$ص = س^{2} - 6س \Leftarrow \frac{دص}{دس} = 2س - 6 \Leftarrow$$

$$\frac{دص}{2س - 6} = دس \text{ .}$$

$$\int \frac{3س - 9}{ص^{2}} \times \frac{دص}{2س - 6} =$$

$$\int 3\cancel{(س - 3)} \times ص^{-2} \times \frac{دص}{2\cancel{(س - 3)}} = \text{.}$$

$$\frac{3}{2}\, \frac{ص^{-2+1}}{-2+1} + جـ = \frac{3}{2}\, \frac{ص^{-1}}{-1} + جـ$$

$$\frac{-3}{2ص} + جـ = \frac{-3}{2(س^{2} - 6س)} + جـ$$

٢) جد قيمة كل من التكاملات الآتية:

أ) $\int \sqrt[3]{(\text{٣س} - ٢)^{٢}}\ \text{ءس}$ ب) $\int (\text{س} - ١)(\text{٢س}^{٢} - \text{٤س} + ١)^{٥}\ \text{ءس}$

جـ) $\int ٢\ \text{قا}^{٢}(٢ - \text{س})\ \text{ءس}$ د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤} + ١)\ \text{ءس}$

الحل

أ) $\int \sqrt[3]{(\text{٣س}-٢)^{٢}}\ \text{ءس} = \int (\text{٣س}-٢)^{\frac{٢}{٣}}\ \text{ءس} = \dfrac{(\text{٣س}-٢)^{\frac{٢}{٣}+١}}{(\frac{٢}{٣}+١)\times ٣} + \text{جـ}$

$= \dfrac{(\text{٣س}-٢)^{\frac{٥}{٣}}}{\cancel{٣}\times\frac{٥}{\cancel{٣}}} + \text{جـ} = \dfrac{١}{٥}\sqrt[3]{(\text{٣س}-٢)^{٥}} + \text{جـ}$

ب) $\int (\text{س}-١)(\text{٢س}^{٢}-\text{٤س}+١)^{٥}\ \text{ءس}$

$\text{ص} = \text{٢س}^{٢}-\text{٤س}+١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س}-٤$

$\dfrac{\text{ءص}}{\text{٤س}-٤} = \text{ءس} \Leftarrow \dfrac{\text{ءص}}{٤(\text{س}-١)} = \text{ءس}$

$\int \cancel{(\text{س}-١)}\ \text{ص}^{٥} \times \dfrac{\text{ءص}}{٤\cancel{(\text{س}-١)}} = \dfrac{١}{٤}\int \text{ص}^{٥}\ \text{ءص}$

$= \dfrac{\text{ص}^{٦}}{٦\times٤} + \text{جـ} = \dfrac{(\text{٢س}^{٢}-\text{٤س}+١)^{٦}}{٢٤} + \text{جـ}$

جـ) $\int ٢\ \text{قا}^{٢}(٢-\text{س})\ \text{ءس} =$

$\dfrac{٢\ \text{ظا}(٢-\text{س})}{-١} + \text{جـ}$

$-٢\ \text{ظا}(٢-\text{س}) + \text{جـ}$

د) $\int \text{٢س}^{٣}\ \text{جا}(\text{س}^{٤}+١)\ \text{ءس}$

$\text{ص} = \text{س}^{٤}+١ \Leftarrow \dfrac{\text{ءص}}{\text{ءس}} = \text{٤س}^{٣} \Leftarrow$

$\dfrac{\text{ءص}}{\text{٤س}^{٣}} = \text{ءس}$

$\int ٢\cancel{\text{س}^{٣}}\ \text{جا ص}\ \dfrac{\text{ءص}}{٤\cancel{\text{س}^{٣}}} =$

$\int \dfrac{١}{٢}\ \text{جا ص ءص} = -\dfrac{١}{٢}\ \text{جتا ص} + \text{جـ}$

$= -\dfrac{١}{٢}\ \text{جتا}(\text{س}^{٤}+١) + \text{جـ}$

٣) احسب قيمة كل من التكاملات الآتية:

أ) $\int_{٠}^{٢} \sqrt{٤\text{س} + ١}\ \text{ءس}$

ب) $\int_{١-}^{١-} ٣\text{س}^{٢}(\text{س}^{٣} - ١)^{٣}\ \text{ءس}$

جـ) $\int_{١}^{٠} ٢\text{س}\ \sqrt[٣]{\text{س}^{٢} - ١}\ \text{ءس}$

د) $\int_{١}^{٢} \frac{٣ - ٢\text{س}}{(\text{س}^{٢} - ٣\text{س})^{٢}}\ \text{ءس}$

الحل

أ) $\int_{٠}^{٢} \sqrt{٤\text{س}+١}\ \text{ءس} = \int_{٠}^{٢} (٤\text{س}+١)^{\frac{١}{٢}} \cdot \text{ءس}$

$$= \left[\frac{(٤\text{س}+١)^{\frac{١}{٢}+١}}{٤ \times (\frac{١}{٢}+١)} \right]_{٠}^{٢} = \left[\frac{(٤\text{س}+١)^{\frac{٣}{٢}}}{٤ \times \frac{٣}{٢}} \right]_{٠}^{٢}$$

$$\left[\frac{\sqrt{(٤\text{س}+١)^{٣}}}{٦} \right]_{٠}^{٢}$$

$$\frac{١}{٦}\left[\sqrt{(٤ \times ٢ + ١)^{٣}} - \sqrt{(٤ \times ٠ + ١)^{٣}} \right]$$

$$\frac{١}{٦}\left(\sqrt{٩^{٣}} - \sqrt{١^{٣}} \right)$$

$$\frac{1}{6}(27-1)$$

$$\frac{1}{6}\times 26 = \frac{13}{3}$$

ب) $\int_{-1}^{1} 3س^{4}(س^{3}-1)^{3}\,دس = صفر$

جـ) $\int_{1}^{0} 2س\sqrt[3]{س^{2}-1}\,.\,دس =$

$$\int_{1}^{0} 2س\,(س^{2}-1)^{\frac{1}{3}}\,دس$$

$$ص = س^{2}-1 \Leftarrow \frac{دص}{دس} = 2س \Leftarrow دس = \frac{دص}{2س}$$

$$\int_{1}^{0} \cancel{2س}\,ص^{\frac{1}{3}}\,\frac{دص}{\cancel{2س}} = \int_{1}^{0} ص^{\frac{1}{3}}\,دص$$

$$\left[\frac{ص^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{1}^{0} = \left[\frac{ص^{\frac{4}{3}}}{\frac{4}{3}}\right]_{1}^{0}$$

$$\frac{3}{4}\left[\sqrt[3]{(س^{2}-1)^{4}}\right]_{1}^{0} = \frac{3}{4}\left(\sqrt[3]{(0^{2}-1)^{4}} - \sqrt[3]{(1-1)^{4}}\right)$$

$$\frac{٣}{٤}\left(\sqrt[٣]{١} - \sqrt[٣]{٠}\right)$$

$$\frac{٣}{٤}\left(١ - ٠\right)$$

$$\frac{٣}{٤} \times ١ = \frac{٣}{٤}$$

د) $\int_{١}^{٢} \frac{٢س-٣}{(س^{٢}-٣س)^{٢}}\, دس = \int_{١}^{٢} (٢س-٣)(س^{٢}-٣س)^{-٢} \cdot دس$

$$ص = س^{٢}-٣س \Leftarrow \frac{دص}{دس} = ٢س-٣ \Leftarrow دس = \frac{دص}{٢س-٣}$$

$$\int_{١}^{٢} \cancel{(٢س-٣)}\, ص^{-٢} \times \frac{دص}{\cancel{٢س-٣}} = \int_{-٢}^{-٢} ص^{-٢}\, دص =$$

$$\left[\frac{-ص^{-٢+١}}{-٢+١}\right]_{١}^{٢} = \left[\frac{-ص^{-١}}{-١}\right]_{١}^{٢} = \left[\frac{١}{ص}\right]_{١}^{٢}$$

$$\left[\frac{١}{س^{٢}-٣س}\right]_{١}^{٢} = \frac{١}{٢^{٢}-٣\times٢} - \frac{١}{١^{٢}-٣\times١} = \frac{١}{-٢} - \frac{١}{-٢}$$

$$= \frac{-١}{٢} + \frac{١}{٢} = \text{صفر}$$

٤) إذا علمت أن ق(−٨) = ٥، ق(٢٧) = −٦، فجد قيمة التكامل الآتي: $\int_{-٢}^{٣} ٣س^{٢}\, قَ(س^{٣})\, دس$

الحل

$$ص = س^{٣} \Leftarrow \frac{دص}{دس} = ٣س^{٢} \Leftarrow دس = \frac{دص}{٣س^{٢}}$$

$$\int_{-٢}^{٣} \cancel{٣س^{٢}}\, قَ(ص) \frac{دص}{\cancel{٣س^{٢}}} = \int_{-٢}^{٣} قَ(ص) \cdot دص =$$

$$\Big[ق(ص)\Big]_{-٢}^{٣} = \Big[ق(س^{٣})\Big]_{-٢}^{٣}$$

$$ق(٣^{٣}) - ق((-٢)^{٣}) = ق(٢٧) - ق(-٨) = -٦ - ٥$$

$$= -١١$$

٥) إذا علمت أن $\int_{5}^{2} \text{ق(س)}\ \text{دس} = 3$، فجد قيمة التكامل الآتي: $\int_{-1}^{2} 8\text{س}\ \text{ق}(\text{س}^2 + 1)\ \text{دس}$

الحل

$\text{ص} = \text{س}^2 + 1 \Leftarrow \dfrac{\text{دص}}{\text{دس}} = 2\text{س} \Leftarrow \dfrac{\text{دص}}{2\text{س}} = \text{دس}$

$\int_{-1}^{2} 8\cancel{\text{س}}\ \text{ق(ص)}\ \dfrac{\text{دص}}{2\cancel{\text{س}}} = \int_{-1}^{2} 4\ \text{ق(ص)}\ \text{دص}$

عندما $\text{س} = -1 \leftarrow \text{ص} = (-1)^2 + 1 = 2$

عندما $\text{س} = 2 \leftarrow \text{ص} = 2^2 + 1 = 5$

$\int_{2}^{5} 4\ \text{ق(ص)}\ \text{دص} = 4\int_{2}^{5} \text{ق(ص)}\ \text{دص} = 4 \times -3 = -12$

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٦) حُلَّ المسألة الواردة في بداية الدرس.

جد قيمة التكامل الآتي:

$$\int_{0}^{4} 2\text{س}\sqrt{\text{س}^2 + 9}\ \text{دس}$$

الحل

$\int_{0}^{4} 2\text{س}\ (\text{س}^2 + 9)^{\frac{1}{2}} \cdot \text{دس}$

$\text{ص} = \text{س}^2 + 9 \Leftarrow \dfrac{\text{دص}}{\text{دس}} = 2\text{س} \Leftarrow$

$\dfrac{\text{دص}}{2\text{س}} = \text{دس}$.

$\int_{0}^{4} \cancel{2\text{س}} \cdot \text{ص}^{\frac{1}{2}} \dfrac{\text{دص}}{\cancel{2\text{س}}} = \int_{0}^{4} \text{ص}^{\frac{1}{2}} \cdot \text{دص} = \left[\dfrac{\text{ص}^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_{0}^{4} = \left[\dfrac{2}{3}\text{ص}^{\frac{3}{2}}\right]_{0}^{4}$

$\dfrac{2}{3}\left[\sqrt{(\text{س}^2 + 9)^3}\right]_{0}^{4}$

$\dfrac{2}{3}\left(\sqrt{(4^2 + 9)^3} - \sqrt{(0 + 9)^3}\right)$

$\dfrac{2}{3}\left(5^3 - 3^3\right) = \dfrac{2}{3}(125 - 27)$

$= \dfrac{2}{3} \times 98 = \dfrac{196}{3}$