

أدرب وأحل المسائل

قاعدة السلسلة الأسئلة (1 - 24)

أجد مشتقة كل اقتران ممّا يأتي:

$$(1) f(x) = e^{4x+2}$$

$$f'(x) = 4e^{4x+2}$$

$$(2) f(x) = 50e^{2x-10}$$

$$f'(x) = 100e^{2x-10}$$

$$(3) f(x) = \cos(x^2 - 3x - 4)$$

$$f'(x) = -(2x - 3) \sin(x^2 - 3x - 4)$$

$$f'(x) = (3 - 2x) \sin(x^2 - 3x - 4)$$

$$(4) f(x) = 10x^2 e^{-x^2}$$

$$f'(x) = (20x) (-2xe^{-x^2}) + (e^{-x^2}) (20x) = 20xe^{-x^2} (1 - x^2)$$

$$(5) f(x) = x + 1/x$$

$$f(x) = x + 1/x = 1 + 1/x$$

$$f'(x) = -1/x^2 + 1/x^2 = -1/x^2 + 1/x^2$$

$$(6) f(x) = x^2 \tan 1/x$$

$$f'(x) = (2x) (\sec^2 1/x) + (\tan 1/x) (-2/x^2)$$

$$f'(x) = -2/x^2 \sec^2 1/x + 2x \tan 1/x$$

$$(7) f(x) = 3x - 5 \cos(\pi x^2)$$

$$f'(x) = 3 + 5(2\pi x) (\pi) \sin(\pi x^2) = 3 + 10\pi^2 x \sin(\pi x^2)$$

$$(8) f(x) = \ln(1 + e^x - e^{-x})$$

$$f(x) = \ln(1 + e^x - e^{-x}) = \ln(1 + e^x) - \ln(1 - e^{-x})$$

$$f'(x) = e^x + e^{-x} + e^x - e^{-x} = 2e^x - e^{-2x}$$

$$(9) f(x) = (\ln x)^4$$

$$f'(x) = 4x (\ln x)^3$$

$$(10) f(x) = \sin x^3 + \sin x^3$$

$$f'(x) = 3x^2 \cos x^3 + \cos x^3 \sin^2 x^3$$

$$(11) f(x) = x^2 + 8x^5$$

$$f'(x) = 2x + 40x^4$$

$$(12) f(x) = 32x^x$$

$$f'(x) = (x)^{2 \ln 3} 32x - 32x^x 2 = (-1 + 2x \ln 3) 32x^x$$

$$(13) f(x) = 2^{-x} \cos \pi x$$

$$f'(x) = (2^{-x}) (-\pi \sin \pi x) + (\cos \pi x) (-\ln 2) 2^{-x}$$

$$= -\pi 2^{-x} \sin \pi x - 2^{-x} (\cos \pi x) \ln 2$$

$$(14) f(x) = 10 \log_4 x^x$$

$$f'(x) = 10x^x \ln 4 - 10 \log_4 x^x 2 = 10 \ln 4 - 10 \log_4 x^x 2$$

$$(15) f(x) = (\sin x + \cos x)^2$$

$$f'(x) = 2 (\sin x + \cos x) (1 + \cos x) (\cos x) - (\sin x) (-\sin x) (1 + \cos x)^2$$

$$= 2x \sin x + \cos x (1 + \cos x)$$

$$= 2 \sin x (1 + \cos x)^2$$

$$(16) f(x) = \log_3 (1 + x \ln x)$$

$$f'(x) = \frac{1}{1 + \ln x (\ln 3)} \left(\ln x + (1 + x \ln x) \cdot \frac{1}{x} \right) = \frac{1 + \ln x (\ln 3) + 1 + x \ln x}{1 + \ln x (\ln 3) (1 + x \ln x)}$$

$$(17) f(x) = e^{\sin 2x} + \sin(e^{2x})$$

$$f'(x) = 2e^{\sin 2x} \cos 2x + 2e^{2x} \cos(e^{2x})$$

$$(18) f(x) = \tan^4(\sec(\cos x))$$

$$f'(x) = 4(\tan(\sec(\cos x)))^3 \sec^2(\sec(\cos x)) \times \sec(\cos x) \tan(\cos x) \times (-\sin x)$$

$$= -4 \tan^3(\sec(\cos x)) \sec^2(\sec(\cos x)) \sec(\cos x) \tan(\cos x) \sin x$$

x أجد معادلة المماس لكل اقتران ممّا يأتي عند قيمة المعطاة:

$$(19) f(x) = 4e^{-0.5x^2}, x = -2$$

$$f(x) = 4e^{-0.5x^2} \quad f(-2) = 4e^{-0.5(-2)^2} = 4e^{-2} \quad f'(x) = -4xe^{-0.5x^2}$$

ميل المماس هو:

$$m = f'(-2) = -4(-2)e^{-0.5(-2)^2} = 8e^{-2}$$

معادلة المماس هي:

$$y - 4e^{-2} = 8e^{-2}(x + 2) \rightarrow y = 8e^{-2}x + 20e^{-2}$$

$$(20) f(x) = x + \cos 2x, x = 0$$

$$f(x) = x + \cos 2x \quad f(0) = 0 + \cos(0) = 1 \quad f'(x) = 1 - 2\sin 2x$$

ميل المماس هو:

$$m = f'(0) = 1 - 2\sin 2(0) = 1$$

معادلة المماس هي:

$$y-1=1(x-0)\rightarrow y=x+1$$

$$(21) f(x)=2x, x=0$$

$$f(x)=2x f(0)=2 \cdot 0=0 \quad f'(x)=(\ln 2)2x$$

ميل المماس هو:

$$m=f'(0)=(\ln 2)2 \cdot 0=0$$

معادلة المماس هي:

$$y-1=(\ln 2)(x-0)\rightarrow y=(\ln 2)x+1$$

$$(22) f(x)=x+1 \sin \pi x^2, x=3$$

$$f(x)=x+1 \sin \pi x^2 \quad f(3)=3+1 \sin 9\pi=3 \quad f'(x)=(x+1)(2\pi \cos \pi x^2)+(\sin \pi x^2)(2x)$$

ميل المماس هو:

$$m=f'(3)=(3+1)(2\pi \cos 9\pi)+(\sin 9\pi)(6)=-14$$

معادلة المماس هي:

$$y+2=-14(x-3)\rightarrow y=-14x+44$$

(23) إذا كان: $(A(x)=f(g(x))$

وكان: $f(-2)=8, f'(-2)=4, f'(5)=3, g(5)=-2, g'(5)=6$ فأوجد $A'(5)$.

$$A'(x)=f'(g(x)) \times g'(x) \quad A'(5)=f'(g(5)) \times g'(5)=f'(-2) \times 6=4 \times 6=24$$

(24) إذا كان: $f(x)=x^2+1$, فأثبت أن $f'(x)=1(x^2+1)^3$.

$$f(x)=x^2+1 \quad f'(x)=(x^2+1)(1)-(x)(2x)=1-x^2 \quad f'(x)=1-x^2$$