



$$\int x dx \quad (7) \sec^4 f$$

$$x \Rightarrow du dx = \sec x) dx u = \tan x (1 + \tan^2 x dx = \int \sec^2 x \times \sec^2 x dx = \int \sec^2 \sec^4 f \\ x = \int (1 + u^2) du = u + \frac{1}{3} u^3 + C = \tan x + \frac{1}{3} \tan^3 x + C = \tan x + \frac{1}{3} \tan^3 x + C$$

$$\int x dx \quad (8) x \cos^2 \tan f$$

$$x \int \tan x \Rightarrow dx = du \sec^2 x \Rightarrow du dx = \sec^2 x dx u = \tan x \sec^2 x dx = \int \tan x \cos^2 \tan f \\ x + C = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} \tan^2 x \times du \sec^2 x dx = \int u \sec^2 x \cos^2 n$$

$$\int x dx \quad (9) \ln \sin f$$

$$u du = -\cos u \times x du = \int \sin x) x dx = \int \sin(\ln x \Rightarrow du dx = \frac{1}{x} \Rightarrow dx = x du \int \sin u = \ln \\ x) + C (\ln u + C = -\cos$$

$$\int x dx \quad (10) x^1 + \sin^2 x \cos \sin f$$

$$x) + C (1 + \sin^2 x dx = \frac{1}{2} \ln x^1 + \sin^2 x \cos x dx = \frac{1}{2} \int 2 \sin x^1 + \sin^2 x \cos \sin f$$

$$\int (2e^x - 2e^{-x})(e^x + e^{-x})^2 dx \quad (11) f$$

$$u = e^x + e^{-x} \Rightarrow du dx = e^x - e^{-x} \Rightarrow dx = du e^x - e^{-x} \int 2e^x - 2e^{-x} (e^x + e^{-x})^2 d \\ x = \int 2(e^x - e^{-x}) u^2 \times du e^x - e^{-x} = \int 2u^2 - 2 du = -\frac{2}{3} u^3 + C = -\frac{2}{3} (e^x + e^{-x})^3 + C$$

$$\int x(x+1)^{x+1} dx \quad (12) - f$$

$$u = x+1 \Rightarrow dx = du, x = u-1 \int -x(x+1)^{x+1} dx = \int 1-u u^u du = \int 1-u u^{3/2} du = \int \\ (u^{3/2} - u^{5/2}) du = \frac{2}{5} u^{5/2} - \frac{2}{7} u^{7/2} + C = \frac{2}{5} (x+1)^{5/2} - \frac{2}{7} (x+1)^{7/2} + C = \\ -\frac{2}{5} x^{5/2} + \frac{2}{5} x^{3/2} - \frac{2}{7} x^{7/2} + \frac{2}{7} x^{5/2} + C$$

$$\int x(x+10)^3 dx \quad (13) f$$

$$u = x+10 \Rightarrow dx = du, x = u-10 \int x(x+10)^3 dx = \int (u-10) u^3 du = \int (u^4 - 10u^3 \\ ) du = \frac{1}{5} u^5 - 10 \frac{1}{4} u^4 + C = \frac{1}{5} (x+10)^5 - 10 \frac{1}{4} (x+10)^4 + C = \frac{1}{5} (x+10)^5 - \frac{5}{2} (x+10)^4 + C$$

$$\int x^2 dx \quad (14) x^2 \tan^7 \sec^2 f$$

$$x^2 dx = \int \sec^2 x \tan^7 x^2 \int \sec^2 x^2 \Rightarrow dx = 2 du \sec^2 x^2 \Rightarrow du dx = 1/2 \sec^2 u = \tan x^2 + C x^2 = 2 \int u^7 du = 14 u^8 + C = 14 \tan^8 x^2 u^7 \times 2 du \sec^2$$

$$(x dx (15 x \sec x + e \sin \sec^3 \int$$

$$x x e \sin x dx + \int \cos x) dx = \int \sec^2 x e \sin x + \cos x dx = \int (\sec^2 x \sec x + e \sin \sec^3 \int x dx + x dx = \int \sec^2 x \sec x + e \sin x \int \sec^3 x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos dx u = \sin x + C x + e \sin x + e u + C = \tan x + \int e u du = \tan x = \tan x e u \times du \cos \int \cos$$

$$(x dx (16 x^3) \cos^3 \sin + 1) \int$$

$$x dx = \int (1 + u^3) \cos^3 x^3) \cos^3 x \int (1 + \sin x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos u = \sin x) du = \int (1 + u^3) (1 - u^2) du = \int (1 + u^3) (1 - \sin^2 x) = \int (1 + u^3) \cos^2 x du \cos) du = \int (1 + u^3) (1 - u^2) du = \int (1 - u^2 + u^3 - u^7) du = u - 1/3 u^3 + 3/4 u^4 - 1/8 u^8 + C x - 3/10 \sin^{10} x + 3/4 \sin^4 x - 1/8 \sin^8 x + C = \sin$$

$$(x dx (17 x \sec^5 \sin \int$$

$$x \int \sin x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin x dx u = \cos x \cos - 5 x dx = \int \sin x \sec^5 \sin \int x + x = - \int u - 5 du = 14 u - 4 + C = 14 \cos - 4 x u - 5 \times du - \sin x dx = \int \sin x \sec^5 n x + C C = 14 \sec^4$$

$$(x dx (18 x \cos^3 x + \tan \sin \int$$

$$x + s x (\sec x \sec x) dx = \int \tan x \sec^3 x + \tan x \sec^2 x dx = \int (\tan x \cos^3 x + \tan \sin \int x dx \cos^3 x + \tan x \int \sin x \sec x \Rightarrow dx = du \tan x \sec x \Rightarrow du dx = \tan x) dx u = \sec^2 x = \int (u + u^2) du = 1/2 u^2 + 1/3 u^3 + C = 1/2 \sec x \sec x (u + u^2) du \tan x \sec x = \int \tan x + C x + 1/3 \sec^3 x$$

أجد قيمة كلا من التكاملات الآتية:

$$(2 x dx (19 x^{1 - \cos 20\pi/4} \sin \int$$

$$|2 x^2 x = |\sin^2 x = \sin^2 \cos^2 - 1$$

لكن الزاوية  $2x$  تكون ضمن الربع الأول عندما  $0 < 2x < \pi/4$

لذا فإن  $2x > 0 \sin$  ويكون  $2x^2 x = \sin^2 \sin$

$$x \Rightarrow x dx u = \sin x \cos 2x dx = \int_0^{\pi/4} 2 \sin 2x \sin 2x dx = \int_0^{\pi/4} 2 \sin^2 x dx = \int_0^{\pi/4} 2(1 - \cos 2x) dx = 2x - \sin 2x \Big|_0^{\pi/4} = \left(\frac{\pi}{2} - 1\right) - (0 - 0) = \frac{\pi}{2} - 1$$

$$(x^2 dx) \int_0^{200\pi/2} x \sin x dx$$

$$x^2 dx = \int_0^{\pi/2} u = x^2 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x} \quad x = \frac{\pi}{2} \Rightarrow u = \frac{\pi^2}{4} \quad x = 0 \Rightarrow u = 0$$

$$\int_0^{\pi/2} x^2 \sin x dx = \int_0^{\pi^2/4} \frac{u}{2} \sin \sqrt{u} \frac{du}{2\sqrt{u}} = \frac{1}{4} \int_0^{\pi^2/4} u \sin \sqrt{u} du$$

$$(01x^3 + 1 + x^2 dx) \int_0^1 (21) dx$$

$$u = 1 + x^2 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x} \quad x^2 = u - 1 \quad x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2$$

$$\int_0^1 (01x^3 + 1 + x^2) dx = \int_1^2 \left(\frac{u-1}{2} + 1 + \frac{u-1}{2}\right) \frac{du}{2\sqrt{u-1}} = \frac{1}{4} \int_1^2 (u-1+2+u-1) du = \frac{1}{4} \int_1^2 (2u) du = \frac{1}{2} \int_1^2 u du = \frac{1}{2} \left(\frac{u^2}{2}\right) \Big|_1^2 = \frac{1}{4} (4 - 1) = \frac{3}{4}$$

$$(x dx) \int_0^{22} x \tan 50\pi/3 \sec^2 x dx$$

$$x \tan x = 0 \Rightarrow u = 0 \quad x = \pi/3 \Rightarrow u = 3 \quad \int_0^{\pi/3} x \tan x \sec^2 x dx = \int_0^3 u \tan \sqrt{u} \sec^2 \sqrt{u} \frac{du}{2\sqrt{u}} = \frac{1}{2} \int_0^3 u \tan \sqrt{u} \sec^2 \sqrt{u} du$$

$$(x-1)e^{(x-1)^2} dx \int_0^2 (23) dx$$

$$u = (x-1)^2 \Rightarrow du dx = 2(x-1) \Rightarrow dx = \frac{du}{2(x-1)} \quad x = 0 \Rightarrow u = 1 \quad x = 2 \Rightarrow u = 1$$

$$\int_0^2 (x-1)e^{(x-1)^2} dx = \int_1^1 \frac{1}{2} e^u du = 0$$

$$(x dx) \int_0^2 (24 + 142) dx$$

$$u = 2 + x \Rightarrow du dx = 1 \Rightarrow dx = du \quad x = 0 \Rightarrow u = 2 \quad x = 2 \Rightarrow u = 4$$

$$\int_0^2 (24 + 142x) dx = \int_2^4 (24 + 142(u-2)) du = \int_2^4 (24 + 142u - 284) du = \int_2^4 (142u - 260) du = 71u^2 - 260u \Big|_2^4 = 71(16 - 4) - 260(4 - 2) = 71(12) - 260(2) = 852 - 520 = 332$$

$$(0110x(1+x^3)^2 dx) \int_0^1 (25) dx$$

$$u = 1 + x^3 \Rightarrow du dx = 3x^2 \Rightarrow dx = \frac{du}{3x^2} \quad x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2$$

$$\int_0^1 10x(1+x^3)^2 dx = \int_1^2 \frac{10}{3} (u-1) du = \frac{10}{3} \left(\frac{u^2}{2} - u\right) \Big|_1^2 = \frac{10}{3} \left(\frac{4}{2} - 2 - \frac{1}{2} + 1\right) = \frac{10}{3} \left(2 - 2 - \frac{1}{2} + 1\right) = \frac{10}{3} \left(\frac{1}{2}\right) = \frac{5}{3}$$

$$(x dx) \int_0^{26} x \sin 0\pi/62 \cos x dx$$

$$x=0 \Rightarrow u=1 \quad x=\pi/6 \Rightarrow u=3/2 \quad \int_0^{\pi/6} 2 \cos x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin u = \cos 2u$$

$$2 \int_{3/2}^2 \cos u du = -2 \ln |x| = -2 \ln |3/2| = -2 \ln 3/2 = -2(\ln 3 - \ln 2) \approx 0.256$$

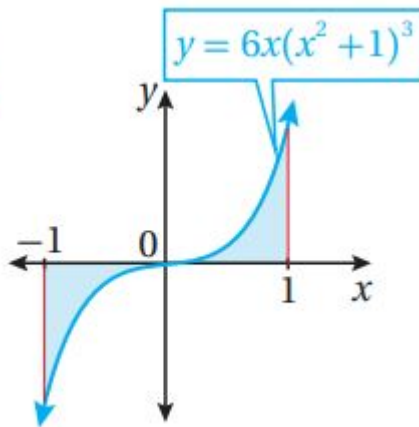
$$\int_0^{\pi/2} x \cot x dx = \int_0^{\pi/2} x \frac{\cos x}{\sin x} dx = \int_0^{\pi/2} x \csc x dx$$

$$x=\pi/2 \Rightarrow u=0 \quad x=\pi/4 \Rightarrow u=1 \quad \int_{\pi/4}^{\pi/2} x \cot x dx \Rightarrow dx = du - \csc^2 x \Rightarrow du dx = -\csc^2 u = \cot u$$

$$x = \int_0^1 (1-u) \cot u du = -\int_0^1 u \cot u du = -\int_0^1 u \frac{\cos u}{\sin u} du = -\int_0^1 u \csc u \cot u du = -\int_0^1 u \csc u \cot u du$$

أجد مساحة المنطقة المظللة في كل من التمثيلات البيانية الآتية:

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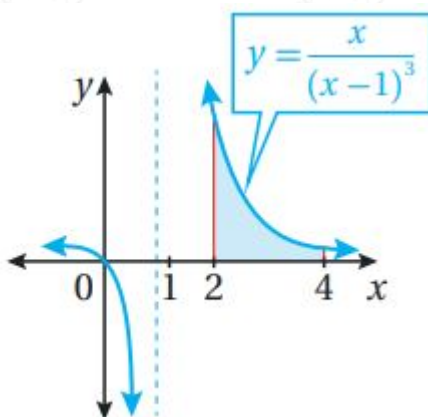
$$A = -\int_{-1}^0 6x(x^2+1)^3 dx + \int_0^1 6x(x^2+1)^3 dx$$

$$u = x^2 + 1 \Rightarrow du dx = 2x \Rightarrow dx = \frac{du}{2x}$$

$$x = -1 \Rightarrow u = 2 \quad x = 0 \Rightarrow u = 1 \quad x = 1 \Rightarrow u = 2$$

$$A = -\int_2^1 3u^3 \frac{du}{u} + \int_1^2 3u^3 \frac{du}{u} = -\int_2^1 3u^2 du + \int_1^2 3u^2 du = -[u^3]_2^1 + [u^3]_1^2 = -(1-8) + (8-1) = 7+7 = 14$$

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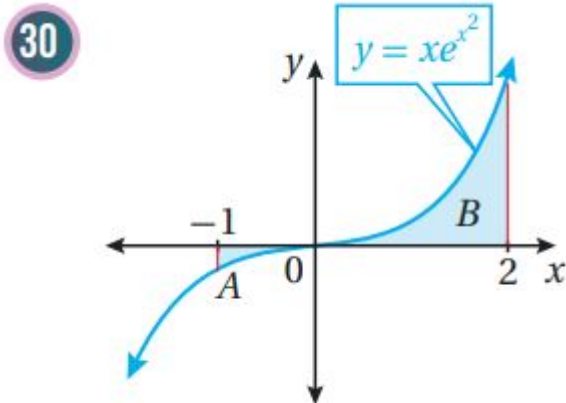


$$A = \int_2^4 \frac{x}{(x-1)^3} dx$$

$$u = x-1 \Rightarrow dx = du, \quad x = u+1$$

$$x=2 \Rightarrow u=1 \quad x=4 \Rightarrow u=3$$

$$A = \int_1^3 \frac{u+1}{u^3} du = \int_1^3 (u^{-2} + u^{-3}) du = (-u^{-1} - \frac{1}{2}u^{-2}) \Big|_1^3 = (-\frac{1}{3} - \frac{1}{18}) - (-1 - \frac{1}{2}) = -\frac{7}{18} + \frac{3}{2} = \frac{-7+27}{18} = \frac{20}{18} = \frac{10}{9}$$

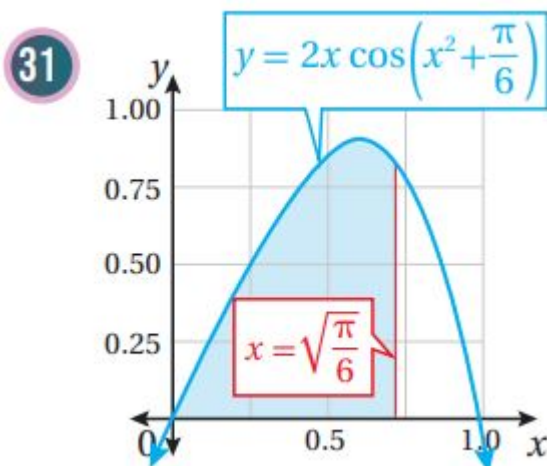


$$u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow \frac{dx}{x} = \frac{du}{2u} \Rightarrow \int \frac{dx}{x} = \frac{1}{2} \int \frac{du}{u} \Rightarrow \ln|x| = \frac{1}{2} \ln|u| \Rightarrow \ln|x| = \frac{1}{2} \ln|x^2| \Rightarrow \ln|x| = \ln|x|$$

$$A = \int_{-1}^0 x e^{x^2} dx = \int_{1}^0 \frac{1}{2} e^u du = \frac{1}{2} [e^u]_{1}^0 = \frac{1}{2} (e^0 - e^1) = \frac{1}{2} (1 - e)$$

$$B = \int_0^2 x e^{x^2} dx = \int_0^4 \frac{1}{2} e^u du = \frac{1}{2} [e^u]_0^4 = \frac{1}{2} (e^4 - e^0) = \frac{1}{2} (e^4 - 1)$$

$$A + B = \frac{1}{2} (1 - e) + \frac{1}{2} (e^4 - 1) = \frac{1}{2} (e^4 - e) \approx 27.658$$



$$u = x^2 + \frac{\pi}{6} \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow \frac{dx}{x} = \frac{du}{2u} \Rightarrow \int \frac{dx}{x} = \frac{1}{2} \int \frac{du}{u} \Rightarrow \ln|x| = \frac{1}{2} \ln|u| \Rightarrow \ln|x| = \frac{1}{2} \ln|x^2 + \frac{\pi}{6}|$$

$$A = \int_0^{\sqrt{\pi/6}} 2x \cos(x^2 + \frac{\pi}{6}) dx = \int_{\pi/6}^{\pi/3} \cos u du = [\sin u]_{\pi/6}^{\pi/3} = \sin(\frac{\pi}{3}) - \sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2} \approx 0.366$$

في كل مما يأتي المشتقة الأولى للاقتران  $(f(x), g(x))$ ، ونقطة يمر بها منحنى  $y = f(x)$ .  
أستعمل المعلومات المعطاة لإيجاد قاعدة الاقتران  $(f(x), g(x))$ :

(32)  $(f(x), g(x)) = (2x(4x^2 - 10)^2, (2, 10))$

$$f(x) = \int f'(x) dx = \int 2x(4x^2 - 10)^2 dx$$

$$u = 4x^2 - 10 \Rightarrow \frac{du}{dx} = 8x \Rightarrow dx = \frac{du}{8x}$$

$$f(x) = \int 2x u^2 \frac{du}{8x} = \frac{1}{4} \int u^2 du = \frac{1}{4} \cdot \frac{1}{3} u^3 + C = \frac{1}{12} (4x^2 - 10)^3 + C$$

$$f(2) = \frac{1}{12} (216) + C = 10 \Rightarrow C = -8 \Rightarrow f(x) = \frac{1}{12} (4x^2 - 10)^3 - 8$$

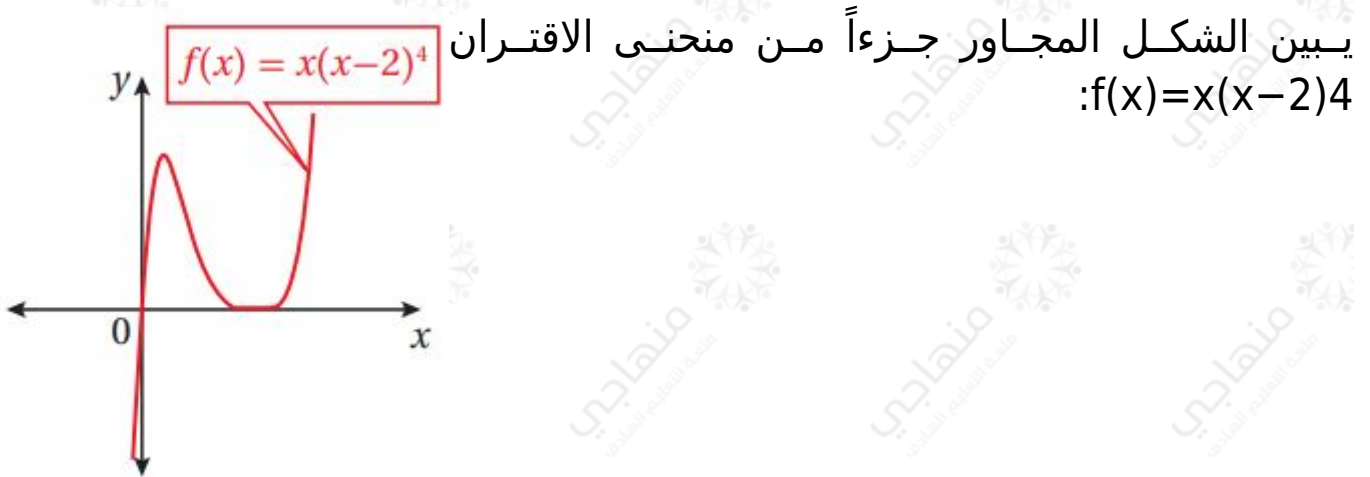
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$$(f'(x) = x^2 e^{-0.2x^3}; (0, 32)) \quad (33)$$

$$f(x) = \int f'(x) dx = \int x^2 e^{-0.2x^3} dx \quad u = -0.2x^3 \Rightarrow du/dx = -0.6x^2 \Rightarrow dx = du / -0.6x^2$$

$$x^2 f(x) = \int x^2 e^u du / -0.6x^2 = -1/0.6 \int e^u du = -5/3 e^u + C \Rightarrow f(x) = -5/3 e^{-0.2x^3} + C$$

$$+ C f(0) = -5/3 + C 32 = -5/3 + C \Rightarrow C = 196 \Rightarrow f(x) = -5/3 e^{-0.2x^3} + 196$$



(34) أجد إحداثي نقطة تماس الاقتران مع المحور x

نجد أصفار الاقتران بحل المعادلة  $f(x) = 0$

$$x(x-2)^4 = 0 \Rightarrow x = 0, x = 2$$

نقطة التقاطع  $(0, 0)$ , فتكون نقطة التماس  $(2, 0)$

ويمكن التحقق بحساب  $f'(2)$ :

$$f'(x) = (x-2)^4 + 4x(x-2)^3 \quad f'(2) = (2-2)^4 + 4(2)(2-2)^3 = 0$$

(35) أجد مساحة المنطقة المحصورة بين منحنى الاقتران  $f(x)$  والمحور x

$$A = \int_0^2 x(x-2)^4 dx \quad u = x-2 \Rightarrow dx = du, x = u+2 \quad x=0 \Rightarrow u = -2 \quad x=2 \Rightarrow u = 0$$

$$A = \int_{-2}^0 (u+2)u^4 du = \int_{-2}^0 (u^5 + 2u^4) du = (1/6 u^6 + 2/5 u^5) \Big|_{-2}^0$$

$$= 0 - (1/6 (-2)^6 + 2/5 (-2)^5) = 32/15$$

(36) يتحرك جسيم في مسار مستقيم، وتعطى سرعته المتجهة بالاقتران:

$\omega t \cos 2v(t) = \sin$  حيث t الزمن بالثواني، و v سرعته المتجهة بالمتري لكل ثانية،



و  $b$  ثابت، إذا انطلق الجسم من نقطة الأصل، فأجد موقعه بعد  $t$  ثانية.

$$wts(t) = wt \Rightarrow dt = du - w \sin wt \Rightarrow dudx = -w \sin wt dt u = \cos wt \cos 2s(t) = \int \sin wt + C wt = -1/w \int u^2 du = -1/3 w u^3 + C \Rightarrow s(t) = -1/30 \cos 3wt u^2 du - w \sin \int \sin$$

لكن  $s(0) = 0$  لأن الجسم انطلق من نقطة الأصل.

$$wt + 13ws(0) = -13w + C0 = -13w + C \Rightarrow C = 13w \Rightarrow s(t) = -13w \cos 3$$



(37) طب: يمثل الاقتران  $C(t)$  تركيز دواء في الدم بعد  $t$  دقيقة من حقنه في جسم مريض، حيث  $C$  مقيسة بالمليغرام لكل سنتيمتر مكعب ( $mg/cm^3$ )، إذا كان تركيز الدواء لحظة حقنه في جسم المريض  $0.5 mg/cm^3$ ، وأخذ يتغير بمعدل  $C'(t) = -0.01e^{-0.01t}(1+e^{-0.01t})^2$ ، فأجد  $C(t)$ .

$$C(t) = \int C'(t) dt = \int -0.01e^{-0.01t}(1+e^{-0.01t})^2 dt u = 1+e^{-0.01t} \Rightarrow du/dt = -0.01e^{-0.01t} \Rightarrow dt = du - 0.01e^{-0.01t} C(t) = \int -0.01e^{-0.01t} u^2 \times du - 0.01e^{-0.01t} = \int u - 2du = -u - 1 + K$$

استعمل الرمز  $K$  لثابت التكامل بدل  $C$  المعتاد لتمييز ثابت التكامل عن رمز الاقتران  $C$ :

$$C(t) = -(1+e^{-0.01t}) - 1 + K C(0) = -(2) - 1 + K12 = -12 \Rightarrow K = 1 \Rightarrow C(t) = -(1+e^{-0.01t}) - 1 + 1 C(t) = -11 + e^{-0.01t} + 1$$

(38) أجد قيمة  $\int 4e^{4x} x e^{-2x} dx$   $3 \ln \ln$ ، ثم اكتب الإجابة بالصيغة الآتية:  $dab + c \ln$ ، حيث  $a, b, c, d$  ثوابت صحيحة.

$$3-2=3-2=1x=|3 \Rightarrow u = e^{\ln u} = e^x - 2 \Rightarrow du/dx = e^x \Rightarrow dx = du e^x e^x = u + 2x = \ln 4e^{4x} x e^{-2x} = \int 1/2 e^{4x} u du e^x = \int 1/2 e^{3x} u dx 3 \ln 4 - 2 = 4 - 2 = 2 \int \ln 4 \Rightarrow u = e^{\ln u} u = \int 1/2 (u+2)^3 u du = \int 1/2 (u^3 + 6u^2 + 12u + 8) u du = \int 1/2 (u^4 + 6u^3 + 12u^2 + 8u) du |u|^{1/2} = (13u^3 + 3u^2 + 12u + 8 \ln$$

(39) إذا كان:  $xf'(x) = \tan$ ، وكان:  $f(3) = 5$ ، فأثبت أن  $\ln |\cos x| + 53 \cos | \cos f(x) = \ln$ .

$$3| + C5 = -\ln |\cos x| + C f(3) = -\ln |\cos x dx = -\ln x \cos x dx = -\int -\sin f(x) = \int \tan$$



$$x|+53\cos|\cos3|=\ln|\cos x|+5+\ln|\cos 3|f(x)=-\ln|\cos 3|+C\Rightarrow C=5+\ln|\cos$$